

CSIR-UGC-NET/JRF | GATE

TIFR|BARC|JEST|IISc|DRDO|ISRO|SLET|ONGC-GT & Ph.D. Entrance

Assignment

Mathematical Physics

PHYSICAL SCIENCES



CAREER ENDEAVOUR
ACADEMY PRIVATE LIMITED

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CAREER ENDEAVOUR

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Best Institute for NET-JRF/GATE & IIT-JAM Exams

Course Structure of MATHEMATICAL PHYSICS

- No. of Questions asked in Part B of CSIR-NET exam : (4-5 ques.) x 3.5 marks each
- No. of Questions asked in Part C of CSIR-NET exam : (4-5 ques.) x 5 marks each
- There will approximately 35-40 marks questions out of 200 marks.
- For the last few years, the qualifying marks for (i) JRF is 90-100 marks (approx.)
(ii) NET is 80-90 marks (approx.)

Course Details:

This course is divided into three parts and there will be a test after completion of each part.

PART I:

- **Vector Algebra & Vector Calculus**
Dot & Cross Product, Triple Product, Vector differentiation, Gradient, Divergence, Curl, Vector Integration, Divergence Theorem, Stokes Theorem, Green's Theorem, Spherical & Cylindrical co-ordinate system.
- **Matrices**
Various types of Matrices, Eigenvalues & Eigenvectors, Cayley Hamilton Theorem, Similarity Transformation, Applications
- **Complex Analysis**
Basics of Complex numbers and Complex functions, C-R equations, Harmonic functions, Milne Thomson Method, Cauchy's Integral Theorem, Taylor Series, Laurent Series, Singular Points, Poles, Residues, Cauchy Residue Theorem, Evaluation of Integrals, Branch points

PART II:

- **Differential Equations & Special Functions**
Separation of variables, Linear first order differential equations, Second order homogeneous and non-homogeneous differential equations, Legendre, Bessel, Hermite & Laguerre Differential Equations.
- **Fourier Series**
Periodic Functions, Dirichlet's Conditions, Fourier series in different forms, Even & Odd Function, Parseval's Formula
- **Fourier Transform & Laplace Transform**
- **Probability**
Elementary idea, Random Variables, Binomial distribution, Poisson distribution, Gaussian (Normal) Distribution, Uniform Distribution, Exponential Distribution, Central Limit Theorem

▪ PART III:

- **Partial Differential Equations**

Laplace, Heat & Wave equations in 2-D & 3-D

- **Numerical Techniques**

Significant figures, Discrete rule of signs, Newton Raphson Method, Bisection Method, Interpolation, Extrapolation, Integration by Trapezoidal rule, Simpson 1/3 rd rule, Runge-Kutta Method, Finite difference Methods.

- **Tensors**

Basic Preview, Einstein Summation Convention, Contra-variant & Covariant Tensor, Levi Civita Tensor, Metric Tensor, Christoffel Symbols

- **Introductory Group Theory**

Basic Preview, Abelian Group, Sub-group, Group of symmetries, Group multiplication Table, Conjugate elements & Classes, Isomorphism, Homomorphism, SU(2) Group, O(3) Group

Reference Books:

- **Mathematical Physics: H. K. Dass, Dr. Rama Verma (S. Chand Publications, 6th Edition)**
- **Advanced Engineering Mathematics: Erwin Kreyszig (Wiley Publications, 9th Edition) -**
- **Vector Analysis: Murray R. Spiegel (Schaum's Series, 2nd Edition)**
- **Functions of Complex Variables: Goyal & Gupta (Pragati Publications).**
- **Laplace & Fourier Transforms: Goyal & Gupta (Pragati Publications)**
- **Probability: Dr. K. C. Sinha (Eduwiser Publishing Group)**
- **Class Notes and Assignments**



CSIR-UGC-NET/JRF | GATE Physics

MATHEMATICAL PHYSICS

Assignment I: Vector Algebra & Vector Analysis

- $\nabla^2 \left(\frac{1}{r} \right)$ is:

(a) 0 (b) $-\delta(r)$ (c) $-4\pi\delta(r)$ (d) $4\pi\delta(r)$
- $\iiint \nabla^2 \left(\frac{1}{r} \right) dV$ $r \neq 0$ is:

(a) 0 (b) -4π (c) 4π (d) 1
- $\text{div}(\hat{i} + \hat{j} + \hat{k})$ is

(a) 3 (b) 2 (c) 1 (d) 0
- The value of t for which three vectors $[(1-t), 0, 0]$, $[1, (1-t), 0]$ & $[1, 1, (1-t)]$ are linearly dependent is

(a) 1 (b) 0 (c) 2 (d) -1
- For what value of n vector $r^n \hat{r}$ is solenoidal.

(a) 0 (b) 1 (c) -1 (d) -3
- Let $\vec{a}$ and $\vec{b}$ be two distinct three-dimensional vectors. Then the component of $\vec{b}$ that is perpendicular to $\vec{a}$ is given by

(a) $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{a^2}$ (b) $\frac{\vec{b} \times (\vec{a} \times \vec{b})}{b^2}$ (c) $\frac{(\vec{a} \cdot \vec{b}) \vec{b}}{b^2}$ (d) $\frac{(\vec{b} \cdot \vec{a}) \vec{a}}{a^2}$
- A vector field is defined everywhere as $\vec{F} = \frac{y^2}{L} \hat{i} + z\hat{k}$. The net flux of $\vec{F}$ associated with a cube of side L , with one vertex at the origin and sides along the positive X , Y , and Z axes, is:

(a) L^3 (b) $4L^3$ (c) $8L^3$ (d) $10L^3$
- If $\vec{r} = x\hat{i} + y\hat{j}$, then:

(a) $\vec{\nabla} \cdot \vec{r} = 0$ and $|\vec{\nabla} \vec{r}| = \vec{r}$ (b) $\vec{\nabla} \cdot \vec{r} = 2$ and $|\vec{\nabla} \vec{r}| = \vec{r}$
 (c) $\vec{\nabla} \cdot \vec{r} = 2$ and $|\vec{\nabla} \vec{r}| = \frac{\vec{r}}{r}$ (d) $\vec{\nabla} \cdot \vec{r} = 3$ and $|\vec{\nabla} \vec{r}| = \frac{\vec{r}}{r}$



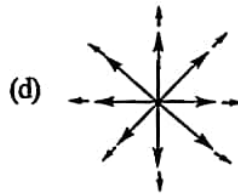
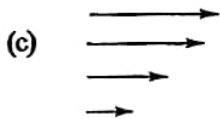
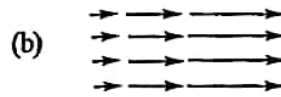
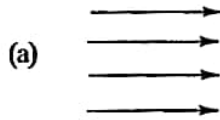
9. Consider a vector $\vec{p} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ in the coordinate system $(\hat{i}, \hat{j}, \hat{k})$. The axes are rotated anti-clockwise about the Y axis by an angle of 60° . The vector $\vec{p}$ in the rotated coordinate system $(\hat{i}', \hat{j}', \hat{k}')$ is:
- (a) $(1-\sqrt{3})\hat{i}' + 3\hat{j}' + (1+\sqrt{3})\hat{k}'$ (b) $(1+\sqrt{3})\hat{i}' + 3\hat{j}' + (1-\sqrt{3})\hat{k}'$
 (c) $(1-\sqrt{3})\hat{i}' + (3+\sqrt{3})\hat{j}' + 2\hat{k}'$ (d) $(1-\sqrt{3})\hat{i}' + (3-\sqrt{3})\hat{j}' + 2\hat{k}'$
10. The value of $\oint \vec{A} \cdot d\vec{\ell}$ along a square loop of side L in a uniform field $\vec{A}$ is:
- (a) 0 (b) $2LA$ (c) $4LA$ (d) L^2A
11. The value of $\oint_S \frac{\vec{r} \cdot d\vec{S}}{r^3}$, where $\vec{r}$ is the position vector and S is a closed surface enclosing the origin, is:
- (a) 0 (b) π (c) 4π (d) 8π
12. If a vector field $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, then $\vec{\nabla} \times (\vec{\nabla} \times \vec{F})$ is
- (a) 0 (b) $\hat{i}$ (c) $2\hat{j}$ (d) $3\hat{k}$
13. Given the four vectors, $u_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $u_2 = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$, $u_3 = \begin{pmatrix} 2 \\ 4 \\ -8 \end{pmatrix}$, $u_4 = \begin{pmatrix} 3 \\ 6 \\ -12 \end{pmatrix}$, the linearly dependent pair is:
- (a) u_1, u_2 (b) u_1, u_3 (c) u_1, u_4 (d) u_3, u_4
14. The unit normal to the curve $x^3y^2 + xy = 17$ at the point (2, 0) is:
- (a) $\frac{(\hat{i} + \hat{j})}{\sqrt{2}}$ (b) $-\hat{i}$ (c) $-\hat{j}$ (d) $\hat{j}$
15. For the function $\phi = x^2y + xy$ the value of $|\vec{\nabla}\phi|$ at $x=y=1$ is
- (a) 5 (b) $\sqrt{5}$ (c) 13 (d) $\sqrt{13}$
- Common data for Q.16 and Q.17.**
- Consider the vector $\vec{V} = \frac{\vec{r}}{r^3}$
16. The surface integral of this vector over the surface of a cube of size and centered at the origin.
- (a) 0 (b) 2π (c) $2\pi a^3$ (d) 4π
17. Which one of the following is NOT correct?
- (a) Value of the line integral of this vector around any closed curve is zero
 (b) This vector can be written as the gradient of some scalar function
 (c) The line integral of this vector from point P to point Q is independent of the path taken.
 (d) This vector can represent the magnetic field of some current distribution
18. The curl of the vector $\vec{A} = z\hat{i} + x\hat{j} + y\hat{k}$ is given by:
- (a) $\hat{i} + \hat{j} + \hat{k}$ (b) $\hat{i} - \hat{j} + \hat{k}$ (c) $\hat{i} + \hat{j} - \hat{k}$ (d) $-\hat{i} - \hat{j} - \hat{k}$



19. The two vectors $\vec{P} = \hat{i}$, $\vec{Q} = (\hat{i} + \hat{j}) / \sqrt{2}$ are:
 (a) related by a rotation (b) related by a reflection through the xy-plane
 (c) related by an inversion (d) not linearly independent
20. A vector $\vec{A} = (5x + 2y)\hat{i} + (3y - z)\hat{j} + (2x - az)\hat{k}$ is solenoidal if the constant a has a value:
 (a) 4 (b) -4 (c) 8 (d) -8
21. Which of the following vectors is orthogonal to the vector $(a\hat{i} + b\hat{j})$, where a and b ($a \neq b$) are constants, and $\hat{i}$ and $\hat{j}$ are unit orthogonal vectors?
 (a) $-b\hat{i} + a\hat{j}$ (b) $-a\hat{i} + b\hat{j}$ (c) $-a\hat{i} - b\hat{j}$ (d) $-b\hat{i} - a\hat{j}$
22. The unit vector normal to the surface $3x^2 + 4y = z$ at the point $(1, 1, 7)$ is:
 (a) $(-6\hat{i} + 4\hat{j} + \hat{k}) / \sqrt{53}$ (b) $(4\hat{i} + 6\hat{j} - \hat{k}) / \sqrt{53}$
 (c) $(6\hat{i} + 4\hat{j} - \hat{k}) / \sqrt{53}$ (d) $(4\hat{i} + 6\hat{j} + \hat{k}) / \sqrt{53}$
23. If S is the closed surface enclosing a volume V and $\hat{n}$ is the unit normal vector to the surface and $\vec{r}$ is the position vector, then the value of the following integral $\oint_S \vec{r} \cdot \hat{n} dS$ is:
 (a) V (b) $2V$ (c) 0 (d) $3V$
24. Consider the set of vectors $\frac{1}{\sqrt{2}}(1, 1, 0)$, $\frac{1}{\sqrt{2}}(0, 1, 1)$ and $\frac{1}{\sqrt{2}}(1, 0, 1)$
 (a) The three vectors are orthonormal (b) The three vectors are linearly independent
 (c) The three vectors cannot form a basis in a three-dimensional real vector space.
 (d) $\frac{1}{\sqrt{2}}(1, 1, 0)$ can be written as the linear combination of $\frac{1}{\sqrt{2}}(0, 1, 1)$ and $\frac{1}{\sqrt{2}}(1, 0, 1)$.
25. If $\vec{A}(t)$ is a vector of constant magnitude, which of the following is true?
 (a) $\frac{d\vec{A}}{dt} = 0$ (b) $\frac{d^2\vec{A}}{dt^2} = 0$ (c) $\frac{d\vec{A}}{dt} \cdot \vec{A} = 0$ (d) $\frac{d\vec{A}}{dt} \times \vec{A} = 0$
26. The function $r^n \hat{r}$ ($r > 0$), where r refers to spherical coordinate system, is:
 (a) an irrotational vector for $n = -2$ (b) a solenoidal vector for $n = -2$
 (c) an irrotational vector for $n = -3$ (d) a solenoidal vector for $n = -3$
27. Two sets of vectors, one with m elements and the other with n elements ($m < n$) span the same linear vector space. If k is the dimension of the vector space, then:
 (a) $k \leq m$ (b) $k \geq m$ (c) $k = m + n$ (d) $k = n$
28. Given $\phi = x^2 - yz$, $\nabla\phi$ at $(3, 4, 1)$ will be:
 (a) $6\hat{i} - 4\hat{j} + \hat{k}$ (b) $6\hat{i} - \hat{j} + 4\hat{k}$ (c) $4\hat{i} - 6\hat{j} - \hat{k}$ (d) $6\hat{i} - 3\hat{j} - 2\hat{k}$



29. Identify the vector field given below which has a finite curl



30. If $\vec{A}$ and $\vec{B}$ are two unit vectors and $\theta \neq 0$ is the angle between them, then

(a) $\sin \theta = \frac{1}{2} |\vec{A} + \vec{B}|$

(b) $\sin \theta = \frac{1}{2} |\vec{A} - \vec{B}|$

(c) $\sin \frac{\theta}{2} = \frac{1}{2} |\vec{A} - \vec{B}|$

(d) $\sin \frac{\theta}{2} = \frac{1}{2} |\vec{A} + \vec{B}|$

31. Given any three non-zero vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$, their triple product $\vec{A} \cdot (\vec{B} \times \vec{C})$ vanishes if

(a) They are perpendicular to each other.

(b) Any two of them are perpendicular

(c) Any two of them are parallel.

(d) They are non-coplanar.

32. The unit vector normal to the surface $x^2 + 4y = z$ at the point $(1, 1, 0)$ is:

(a) $\frac{1}{\sqrt{21}}(2\hat{i} + 4\hat{j} - \hat{k})$ (b) $\frac{1}{\sqrt{21}}(-2\hat{i} + 4\hat{j} - \hat{k})$ (c) $\frac{1}{\sqrt{21}}(2\hat{i} - 4\hat{j} - \hat{k})$ (d) $\frac{1}{\sqrt{21}}(2\hat{i} + 4\hat{j} + \hat{k})$

33. The necessary and sufficient condition that $\oint_C \vec{A} \cdot d\vec{r} = 0$, for any closed curve C is

(a) $\vec{\nabla} \cdot \vec{A} = 0$

(b) $\vec{\nabla} \times \vec{A} = 0$

(c) $\vec{\nabla} \cdot \vec{\nabla} \times \vec{A} = 0$

(d) $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = 0$

34. Any arbitrary vector in a three dimensional Cartesian space can be expressed as a linear combination of the following number of linearly independent vectors:

(a) Arbitrary number

(b) 1

(c) 2

(d) 3

35. Which one of the following vectors is normal to the surface $x^2y + z^3 + xz^2 = 10$?

(a) $x^2y\hat{i} + z^3\hat{j} + xz^2\hat{k}$

(b) $2xy\hat{i} + 2xz\hat{k}$

(c) $2y\hat{i} + (6z + 2x)\hat{k}$

(d) $(2xy + z^2)\hat{i} + x^2\hat{j} + (3z^2 + 2xz)\hat{k}$

36. The line integral of $\vec{A}$ vanishes about every closed path. Then $\vec{A}$ must be equal to

(a) Curl of a vector function

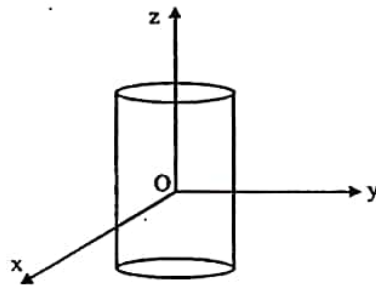
(b) Gradient of a scalar function

(c) Gradient of a vector function

(d) Zero



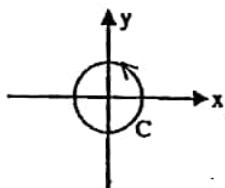
37. When the fluid is incompressible, the equation of continuity can be reduced to $\frac{\partial \rho}{\partial t} = 0 = \rho (\vec{\nabla} \cdot \vec{v})$. Since the density ρ is constant in this case. Here $\vec{v}$ is the velocity of a typical particle of the fluid. Further, if the flow is irrotational, then the equation can be rewritten as
- (a) $\vec{\nabla} \cdot \vec{\nabla} \phi = 0$ (b) $\vec{\nabla} \times \vec{\nabla} \phi = 0$ (c) $\vec{\nabla} \phi = 0$ (d) $\vec{v} = \text{constant}$
38. The vector perpendicular to $3\hat{i} + 4\hat{j} - 5\hat{k}$ is:
- (a) $-2\hat{i} + 4\hat{j} - 2\hat{k}$ (b) $2\hat{i} + 4\hat{j} - 2\hat{k}$ (c) $2\hat{i} - 4\hat{j} + 2\hat{k}$ (d) $2\hat{i} - 4\hat{j} - 2\hat{k}$
39. The unit vector normal to the surface $x^2 + y^2 - z = 1$ at the point P(1, 1, 1) is:
- (a) $\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$ (b) $\frac{2\hat{i} + \hat{j} - \hat{k}}{\sqrt{6}}$ (c) $\frac{\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{6}}$ (d) $\frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$
40. Consider a cylinder of height h and radius a , closed at both ends, centered at the origin. Let $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ be the position vector and $\hat{n}$ a unit vector normal to the surface. The surface integral $\int_S \vec{r} \cdot \hat{n} ds$ over the closed surface of the cylinder is:



- (a) $2\pi a^2(a+h)$ (b) $3\pi a^2 h$ (c) $2\pi a^2 h$ (d) Zero.
41. Given the vector $\vec{A}(y, -x, 0)$, the line integral $\oint_C \vec{A} \cdot d\vec{l}$, where C is a circle of radius 5 units with its centre at the origin, (correct to the first decimal place) is
- (a) 172.8 (b) 157.1 (c) -146.3 (d) 62.8
42. The value of the integral $I = \int_S \vec{r} \cdot d\vec{s}$ where S is the surface enclosing the volume V is
- (a) 3 (b) V (c) 3V (d) 0
43. The cross product of $(\hat{i} + \hat{j}) \times (\hat{j} + \hat{k})$ where $\hat{i}, \hat{j}, \hat{k}$ are orthonormal unit vectors in 3-dimensions, is
- (a) $\hat{i} - \hat{k}$ (b) $\hat{i} + \hat{k}$ (c) $\hat{i} + \hat{j} + \hat{k}$ (d) $\hat{k} - \hat{j} + \hat{i}$



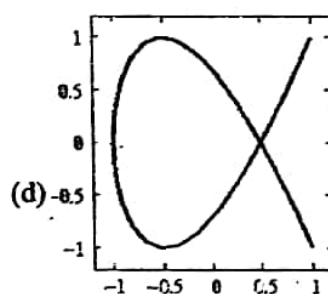
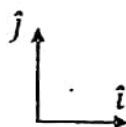
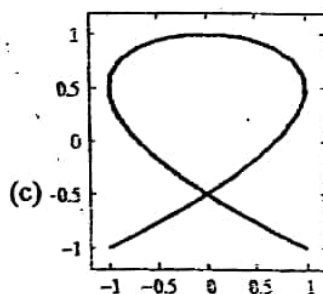
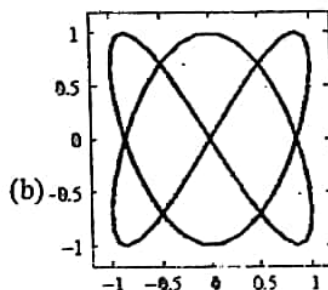
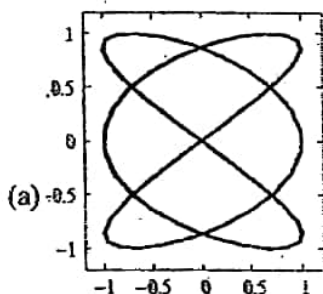
44. Identify the CORRECT statement for the following vectors $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$
- (a) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent
 (b) The vectors $\vec{a}$ and $\vec{b}$ are linearly dependent
 (c) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal
 (d) The vectors $\vec{a}$ and $\vec{b}$ are normalized
45. Given $\vec{F} = \vec{r} \times \vec{B}$, where $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})$ is a constant vector and $\vec{r}$ is the position vector. The value of $\oint_C \vec{F} \cdot d\vec{r}$, where C is a circle of unit radius centered at origin is,



- (a) 0 (b) $2\pi B_0$ (c) $-2\pi B_0$ (d) 1
46. A vector perpendicular to any vector that lies on the plane defined by $x + y + z = 5$, is
- (a) $\hat{i} + \hat{j}$ (b) $\hat{j} + \hat{k}$ (c) $\hat{i} + \hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{j} + 5\hat{k}$
47. A two-dimensional vector

$$\vec{A}(t) = \hat{i} \sin 2t + \hat{j} \cos 3t$$

Which of the following graphs best describes the locus of the tip of the vector, as 't' is varied from 0 to 2π ?



48. Consider the surface corresponding to the equation

$$4x^2 + y^2 + z = 0$$

A possible unit tangent to this surface at the point (1, 2, -8) is

- (a) $\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}$ (b) $\frac{1}{5}\hat{j} - \frac{4}{5}\hat{k}$ (c) $\frac{4}{9}\hat{i} - \frac{8}{9}\hat{j} + \frac{1}{9}\hat{k}$

49. The unit normal vector at the point $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, is

- (a) $\frac{bc\hat{i} + ca\hat{j} + ab\hat{k}}{\sqrt{a^2b^2 + b^2c^2 + c^2a^2}}$ (b) $\frac{a\hat{i} + b\hat{j} + c\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$ (c) $\frac{b\hat{i} + c\hat{j} + a\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$ (d) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

50. If $\vec{A} = \hat{i}yz + \hat{j}xz + \hat{k}xy$, then the integral $\oint_C \vec{A} \cdot d\vec{\ell}$ (where C is along the perimeter of a rectangular area bounded by $x=0$, $x=a$ and $y=0$, $y=b$) is

- (a) $\frac{1}{2}(a^3 + b^3)$ (b) $\pi(ab^2 + a^2b)$ (c) $\pi(a^3 + b^3)$ (d) 0

51. If $\vec{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and C is the circle of unit radius in the plane defined by $z=1$, with the centre on the z -axis, then the value of the integral $\oint_C \vec{A} \cdot d\vec{\ell}$ is

- (a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

Vector Algebra and Vector Analysis Answer Key

- | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (c) | 2. (a) | 3. (d) | 4. (a) | 5. (d) | 6. (a) | 7. (a) |
| 8. (c) | 9. (a) | 10. (a) | 11. (c) | 12. (a) | 13. (d) | 14. (d) |
| 15. (d) | 16. (d) | 17. (d) | 18. (a) | 19. (a) | 20. (c) | 21. (a) |
| 22. (c) | 23. (d) | 24. (b) | 25. (c) | 26. (b) | 27. (b) | 28. (b) |
| 29. (c) | 30. (c) | 31. (c) | 32. (a) | 33. (b) | 34. (d) | 35. (d) |
| 36. (b) | 37. (b) | 38. (d) | 39. (d) | 40. (b) | 41. (b) | 42. (c) |
| 43. (d) | 44. (a) | 45. (c) | 46. (c) | 47. (b) | 48. (a) | 49. (a) |
| 50. (d) | 51. (d) | | | | | |





CSIR-UGC-NET/JRF | GATE Physics

MATHEMATICAL PHYSICS

Assignment II: Matrix Algebra

- The inverse of matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ is:
 (a) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (b) $\frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ (c) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ (d) does not exist
- Which one of the following is not hermitian?
 (a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}$
- Eigen values of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are
 (a) 2, 1, 0 (b) 1, 1, 1 (c) 3, 0, 0 (d) 1, 2, 1
- The eigenvalues of a matrix are i , $-2i$ and $3i$. The matrix is:
 (a) unitary (b) anti-unitary (c) hermitian (d) anti-hermitian
- The eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are:
 (a) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
 (c) 6, 1 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (d) 2, 5 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$
- The trace of a 3×3 matrix is 2. Two of its eigen values are 1 and 2. The third eigenvalue is:
 (a) -1 (b) 0 (c) 1 (d) 2
- A linear transformation T , defined as $T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \end{pmatrix}$, transform a vector $\vec{x}$ from a three-dimensional real space to a two-dimensional real space. The transformation matrix T is:
 (a) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$



Common data for Q. 8 and Q. 9:

One of the eigen values of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is 5.

8. The other two eigen values are:
 (a) 0 and 0 (b) 1 and 1 (c) 1 and -1 (d) -1 and -1
9. The normalized eigen vector corresponding to the eigen value 5 is:
 (a) $\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$ (b) $\frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (c) $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (d) $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$
10. Eigen values of the matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{pmatrix}$ are:
 (a) -2, -1, 1, 2 (b) -1, 1, 0, 2 (c) 1, 0, 2, 3 (d) -1, 1, 0, 3
11. The determinant of a 3×3 real symmetric matrix is 36. If two of its eigen values are 2 and 3 then the third eigenvalue is:
 (a) 4 (b) 6 (c) 8 (d) 9
12. A real traceless 4×4 unitary matrix has two eigen values -1 and 1. The other eigenvalues are:
 (a) zero and +2 (b) -1 and +1 (c) zero and +1 (d) +1 and +1
13. The eigenvalues of the matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$ are:
 (a) +1 and +1 (b) zero and +1 (c) zero and +2 (d) -1 and +1
14. A 3×3 matrix has eigenvalues 0, $2 + i$ and $2 - i$. Which one of the following statements is correct?
 (a) The matrix is Hermitian (b) The matrix is unitary
 (c) The inverse of the matrix exists (d) The determinant of the matrix is zero
15. If two matrices A and B can be diagonalized simultaneously, which of the following is true?
 (a) $A^2B = B^2A$ (b) $A^2B^2 = B^2A$ (c) $AB = BA$ (d) $AB^2AB = BABA^2$
16. Which one of the following matrices is the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$?
 (a) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$
17. For any operator A, $i(A^\dagger - A)$ is:
 (a) Hermitian (b) Anti-Hermitian (c) Unitary (d) Orthogonal
18. A square matrix A is unitary if:
 (a) $A^\dagger = A$ (b) $A^\dagger = A^{-1}$ (c) $\text{Tr}(A) = 1$ (d) $\det(A) = 1$



19. If A is an antisymmetric matrix and A^T be the transpose of this matrix, then which one of the following relations does not hold good?
 (a) $A^T = -A$ (b) $AA^T = A^T A$
 (c) A^2 is an antisymmetric matrix (d) A^2 is a symmetric matrix
20. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ are given by λ_1, λ_2 and λ_3 . Which one of the following statement is NOT TRUE?
 (a) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (b) $\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_3 \lambda_1 = 0$
 (c) All eigenvalues are real (d) Product of the eigenvalues is 1
21. The matrix $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is:
 (a) orthogonal (b) hermitian (c) symmetric (d) anti-symmetric
22. A given $(n \times n)$ nilpotent matrix A satisfies the equation $A^k = 0$ for $1 < k < n$. Therefore,
 (a) Exactly k eigenvalues of A must be zero. (b) Exactly $(n-k)$ eigenvalue of A must be zero.
 (c) Every eigenvalue of A is zero. (d) A can have $(n-1)$ non-zero eigenvalues.
23. Consider the $(n \times n)$ matrix A with every element equal to unity. Which of the following statement is correct?
 (a) The eigenvalues of A are all equal to unity. (b) All the eigenvalues of A are zero.
 (c) The largest eigenvalue of A is n . (d) A cannot be diagonalised by a similarity transformation.
24. The eigenvalues of the matrix $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ are
 (a) 0, 0 (b) 1, 0 (c) 1, -1 (d) 1, 1
25. If $L(x)$ is a linear differential operator and $y_1(x), y_2(x)$ are two arbitrary functions. Then
 (a) $L(x)(y_1 + y_2) = Ly_1 + Ly_2$ (b) $L(x)(y_1 + y_2) = y_1 Ly_2 + y_2 Ly_1$
 (c) $L(x)(y_1 y_2) = Ly_1 + Ly_2$ (d) $L(x)(y_1 - y_2) = Ly_1 + Ly_2$
26. For which of the following values of a , the inverse of the matrix, $M = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 3 & 5 \\ 0 & 4 & a \end{pmatrix}$, does not exist?
 (a) -4 (b) 4 (c) 0 (d) 1
27. The eigenvalues of the matrix $M = \begin{pmatrix} 1 & 0 \\ 1 & i \end{pmatrix}$ are
 (a) 1, 1 (b) 1, i (c) 1, 0 (d) Cannot be determined.



28. Which one of the following matrices is orthogonal?

(a) $\begin{pmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

(b) $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

(c) $\begin{pmatrix} -\cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

(d) $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & -\cos \theta \end{pmatrix}$

29. The eigenvalues of a skew-Hermitian matrix are

(a) Real and positive

(b) Real and negative

(c) Either zero or pure imaginary

(d) Complex with nonzero real parts.

30. The number of independent real parameters of a most general hermitian matrix of order 4 is:

(a) 4

(b) 8

(c) 16

(d) 32

31. The eigen values of the matrix, $\begin{bmatrix} 0 & 0 & -i \\ 0 & 2 & 0 \\ i & 0 & 0 \end{bmatrix}$ are

(a) Purely imaginary

(b) Complex

(c) Real

(d) Zero

32. If a matrix A satisfies the condition $A^2 = I$ and $\text{Tr}(A) = 0$ (where I is a $n \times n$ identity matrix), then

(a) The determinant of A must be 0

(b) The determinant of A must be +1

(c) A must be odd dimensional

(d) A must be even dimensional

33. Two matrices A and B are said to be similar if $B = P^{-1}AP$ for some invertible matrix P. Which of the following statements is NOT TRUE?

(a) $\text{Det } A = \text{Det } B$

(b) $\text{Trace of } A = \text{Trace of } B$

(c) A and B have the same eigenvectors

(d) A and B have the same eigenvalues.

34. A 3×3 matrix has elements such that its trace is 11 and its determinant is 36. The eigenvalues of the matrix are all known to be positive integers. The largest eigenvalue of the matrix is:

(a) 18

(b) 12

(c) 9

(d) 6

35. The matrix A, defined by $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & a & b \\ 0 & -b & a \end{pmatrix}$ is orthogonal if

(a) $a = 1, b = -1$

(b) $a = 1/\sqrt{2}, b = -1/\sqrt{2}$

(c) $a = 1/\sqrt{2}, b = -i/\sqrt{2}$

(d) $a = 1, b = 1$

36. Consider the matrix $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

A. The eigenvalues of M are

(a) 0, 1, 2

(b) 0, 0, 3

(c) 1, 1, 1

(d) -1, 1, 3



B. The exponential of M simplifies to (I is the 3×3 identity matrix)

- (a) $e^M = I + \left(\frac{e^3 - 1}{3}\right)M$ (b) $e^M = I + M + \frac{M^2}{2!}$
 (c) $e^M = I + 3^3 M$ (d) $e^M = (e - 1)M$

37. The eigenvalues of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are

- (a) 3, 0, 1 (b) 3, 0, 0 (c) 1, 2, 0 (d) 1, 1, 1

38. If A and B are two $n \times n$ matrices, then the trace of $C = [A, B]$ is

- (a) $\text{Tr}(AB)$ (b) $\text{Tr}(A) \text{Tr}(B)$ (c) 0 (d) $\text{Tr}(A) + \text{Tr}(B)$

39. Consider the matrix $M = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$ where a and c are real and b may be complex. If M can be diagonalized by a matrix S , then S is

- (a) diagonal (b) symmetric (c) orthogonal (d) unitary

40. The trace and the determinant of a 2×2 matrix are denoted by T and D . The eigenvalues of the matrix are given by

- (a) $\frac{T \pm \sqrt{T^2 - 4D}}{2}$ (b) $\pm \frac{\sqrt{T^2 - 4D}}{2}$ (c) $\frac{T \pm D}{2}$ (d) $\pm \frac{T}{2}$

41. The eigenvalues of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are

- (a) 0, 1, 1 (b) $0, -\sqrt{2}, \sqrt{2}$ (c) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (d) $\sqrt{2}, \sqrt{2}, 0$

42. For an $N \times N$ matrix consisting of all ones,

- (a) all eigenvalues = 1 (b) all eigenvalues = 0
 (c) the eigenvalues are $1, 2, \dots, N$ (d) one eigenvalue = N , the others = 0

43. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$

- (a) (1, 4, 9) (b) (0, 7, 7) (c) (0, 1, 13) (d) (0, 0, 14)

44. The eigenvalues of the antisymmetric matrix, $A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$ where n_1, n_2 and n_3 are the components of a unit vector, are

- (a) 0, i , $-i$ (b) 0, 1, -1 (c) 0, $1 + i$, $-1, -i$ (d) 0, 0, 0



45. A 2×2 matrix A has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of 'n' such that $A^n = 1$ is:
 (a) 20 (b) 30 (c) 60 (d) 120
46. Given a 2×2 unitary matrix U satisfying $U^\dagger U = UU^\dagger = 1$ with $\det U = e^{i\varphi}$, one can construct a unitary matrix V ($V^\dagger V = VV^\dagger = 1$) with $\det V = 1$ from it by
 (a) Multiplying U by $e^{-i\varphi/2}$
 (b) Multiplying any single element of U by $e^{-i\varphi}$
 (c) Multiplying any row or column of U by $e^{-i\varphi/2}$
 (d) Multiplying U by $e^{-i\varphi}$
47. Consider an $n \times n$ ($n > 1$) matrix A , in which A_{ij} is the product of the indices i and j (namely $A_{ij} = ij$). The matrix A
 (a) has one degenerate eigenvalue with degeneracy $(n-1)$
 (b) has two degenerate eigenvalues with degeneracies 2 and $(n-2)$
 (c) has one degenerate eigenvalue with degeneracy n
 (d) does not have any degenerate eigenvalue
48. The product MN of two Hermitian matrices M and N is anti-Hermitian. It follows that
 (a) $\{M, N\} = 0$ (b) $[M, N] = 0$ (c) $M^\dagger = N$ (d) $M^\dagger = N^{-1}$
49. The coordinate transformation $x' = 0.8x + 0.6y$, $y' = 0.6x - 0.8y$ represents
 (a) a translation (b) a proper rotation
 (c) a reflection (d) none of the above
50. A 3×3 matrix M has $\text{Tr}[M] = 6$, $\text{Tr}[M^2] = 26$ and $\text{Tr}[M^3] = 90$. Which of the following can be a possible set of eigenvalues of M ?
 (a) $\{1, 1, 4\}$ (b) $\{-1, 0, 7\}$ (c) $\{-1, 3, 4\}$ (d) $\{2, 2, 2\}$
51. The eigenvalues of the matrix $\begin{pmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ are
 (a) 5, 2, -2 (b) -5, -1, -1 (c) 5, 1, -1 (d) -5, 1, 1
52. Consider the matrix $M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$
 The eigenvalues of M are
 (a) -5, -2, 7 (b) -7, 0, 7 (c) -4i, 2i, 2i (d) 2, 3, 6



53. The matrices

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

satisfy the commutation relations

(a) $[A, B] = B + C, [B, C] = 0, [C, A] = B + C$

(b) $[A, B] = C, [B, C] = A, [C, A] = B$

(c) $[A, B] = B, [B, C] = 0, [C, A] = A$

(d) $[A, B] = C, [B, C] = 0, [C, A] = B$

Matrix Algebra Answer Key

- | | | | | | | |
|------------|---------|---------|---------|---------|---------|---------|
| 1. (b) | 2. (d) | 3. (c) | 4. (d) | 5. (a) | 6. (a) | 7. (a) |
| 8. (c) | 9. (d) | 10. (a) | 11. (b) | 12. (b) | 13. (c) | 14. (d) |
| 15. (c) | 16. (c) | 17. (a) | 18. (a) | 19. (c) | 20. (c) | 21. (a) |
| 22. (c) | 23. (c) | 24. (c) | 25. (a) | 26. (b) | 27. (b) | 28. (b) |
| 29. (c) | 30. (a) | 31. (c) | 32. (d) | 33. (c) | 34. (d) | 35. (b) |
| 36. (b, c) | 37. (b) | 38. (c) | 39. (d) | 40. (a) | 41. (b) | 42. (d) |
| 43. (d) | 44. (a) | 45. (c) | 46. (a) | 47. (a) | 48. (a) | 49. (c) |
| 50. (c) | 51. (c) | 52. (b) | 53. (d) | | | |





CSIR-UGC-NET/JRF | GATE Physics

MATHEMATICAL PHYSICS

Assignment III: Complex Analysis

- The residue of $\frac{1}{1+z^2}$ at $z=i$ is :
(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{2i}$ (d) $\frac{1}{-2i}$
- The value of $\oint_C \frac{dz}{(z-3)}$ if C is the circle $|z-2|=5$ is
(a) πi (b) $2\pi i$ (c) 0 (d) 2π
- The complex function $f(z) = e^{\frac{+1}{(z-1)^2}}$ has singular point at $z=1$ which is:
(a) a pole of order 1 (b) a pole of order 2
(c) an isolated essential singularity (d) a non-isolated essential singularity
- The contour integral $\oint \frac{dz}{z^2 + a^2}$ is to be evaluated on a circle of radius $2a$ centered at the origin. It will have contributions only from the points.
(a) $\frac{1+i}{\sqrt{2}}a$ and $-\frac{1+i}{\sqrt{2}}a$ (b) ia and $-ia$
(c) $ia, -ia, \frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$ (d) $\frac{1+i}{\sqrt{2}}a, -\frac{1+i}{\sqrt{2}}a, \frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$
- The value of $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, where C is a circle defined by $|z|=3$, is:
(a) $\frac{8\pi i}{3} e^{-2}$ (b) $\frac{8\pi i}{3} e^{-1}$ (c) $\frac{8\pi i}{3} e$ (d) $\frac{8\pi i}{3} e^2$
- Which of the following functions of the complex variable z is NOT analytic everywhere?
(a) e^z (b) $|z|^3$ (c) z^3 (d) $\frac{|z|}{3}$
- The value of the integral $\oint_C \frac{dz}{z+3}$ where C is a circle (anticlockwise) with $|z|=4$, is:
(a) 0 (b) πi (c) $2\pi i$ (d) $4\pi i$
- The inverse of the complex number $\frac{3+4i}{3-4i}$ is:
(a) $\frac{7}{25} + i\frac{24}{25}$ (b) $-\frac{7}{25} + i\frac{24}{25}$ (c) $\frac{7}{25} - i\frac{24}{25}$ (d) $-\frac{7}{25} - i\frac{24}{25}$



9. The value of $\int_C \frac{dz}{(z^2 + a^2)}$, where C is a unit circle (anti clockwise) centered at the origin in the complex z-plane is:
- (a) π for $a = 2$ (b) zero for $a = \frac{1}{2}$ (c) 4π for $a = 2$ (d) $\frac{\pi}{2}$ for $a = \frac{1}{2}$
10. The value of the integral $\int_C dz/z^2$, where z is a complex variable and C is the unit circle with the origin as its centre, is:
- (a) 0 (b) $2\pi i$ (c) $4\pi i$ (d) $-4\pi i$
11. If a function $f(z) = u(x, y) + iv(x, y)$ of the complex variable $z = x + iy$, where x, y, u and v are real, is analytic in a domain D of z, then which of the following is true?
- (a) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ (b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$
- (c) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial y}$ (d) $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 v}{\partial x \partial y}$
12. The value of the integral $\int_C z^{10} dz$, where C is the unit circle with the origin as the centre is:
- (a) 0 (b) $z^{11}/11$ (c) $2\pi iz^{11}/11$ (d) $1/11$
13. If Z_1 and Z_2 are complex numbers, which of the following is always true?
- (a) $Z_1 * Z_2 + Z_2 * Z_1 = 2 Z_1 * Z_2$ (b) $Z_1 * Z_2 + Z_2 * Z_1 = 2 Z_1 Z_2$
- (c) $Z_1 * Z_2 + Z_2 * Z_1 \leq 2 |Z_1| |Z_2|$ (d) $Z_1 * Z_2 + Z_2 * Z_1 \geq 2 |Z_1| |Z_2|$
14. Which of the following inequalities are valid in case of complex numbers?
- (a) $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$ (b) $|Z_1 + Z_2| > |Z_1| + |Z_2|$
- (c) $|Z_1 - Z_2| \geq |Z_1| - |Z_2|$ (d) $|Z_1 - Z_2| < |Z_1| - |Z_2|$
15. The function $\tan(1/z)$ does not have a pole at $z =$
- (a) $\pi/2$ (b) $2/\pi$ (c) $2/(3\pi)$ (d) $-2/(3\pi)$
16. The real part of a complex function $f(z)$, analytic in a region is given by $u(x, y) = x^2 - y^2$. If the function vanishes at $z = 0$, the imaginary part of the function is
- (a) $2xy$ (b) $-2xy$ (c) $x^2 + y^2$ (d) $y^2 - x^2$
17. The residue of the function, $f(z) = \frac{z}{(2z+1)(5-z)}$ at $z = -1/2$ is:
- (a) $1/11$ (b) $-1/11$ (c) $1/22$ (d) $-1/22$
18. If z_1 and z_2 are two complex numbers then:
- (a) $|z_1 + z_2| < |z_1| + |z_2|$ (b) $|z_1 + z_2| = |z_1| + |z_2|$
- (c) $(z_1 + z_2)^2 > 0$ (d) $|z_1 + z_2|^2 = (z_1 + z_2)(z_1 + z_2)$
19. Two complex numbers Z_1 and Z_2 are given to satisfy $|Z_1| = |Z_2|$ necessarily follows that:
- (a) $Z_1 = Z_2$ (b) $Z_1 = e^{i\theta} Z_2$ (c) $Z_1 = Z_2$ (d) $Z_1 = Z_2^{-1}$
20. The continuous complex function $f(z) = z^* = x - iy$ is differentiable
- (a) only at $z = 0$ (b) only at $z = 1$ (c) nowhere (d) everywhere

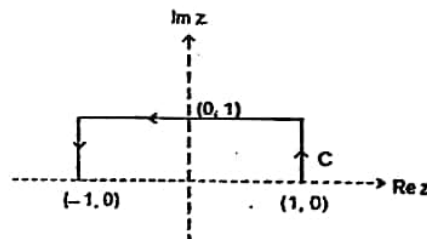


21. Two functions of a complex variable $z = x + iy$ are given as (i) $|z|$ and (ii) z^2 . which of the following statements is correct?
 (a) Both (i) and (ii) are analytic (b) (i) is analytic and (ii) is nonanalytic
 (c) Both are nonanalytic (d) (i) is nonanalytic and (ii) is analytic
22. A function $f(Z)$ of a complex variable Z is given as $f(Z) = Z \exp\left(\frac{1}{Z}\right)$. At $Z = 0$.
 (a) It has no singularity
 (b) It has a simple pole and residue is 1
 (c) It has an essential singularity and residue is $1/2$
 (d) It has a pole of order 2 and residue is $1/6$
23. A function $f(Z)$ of a complex variable Z is analytic everywhere in the complex plane except at $Z = Z_0$. Two contours C_1 and C_2 around Z_0 are shown in the figure.



The values of the integrals $I_1 = \oint_{C_1} \frac{f(Z)dZ}{Z - Z_0}$, $I_2 = \oint_{C_2} \frac{f(Z)dZ}{Z - Z_0}$ are

- (a) $I_1 = 0, I_2 = 2\pi i f(Z_0)$ (b) $I_1 = 2\pi i f(Z_0), I_2 = 0$
 (c) $I_1 = \pi i f(Z_0), I_2 = \pi i f(Z_0)$ (d) $I_1 = 2\pi i f(Z_0), I_2 = 2\pi i f(Z_0)$
24. The value of the integral $\int_C dz z^2 e^z$, where C is an open contour in the complex z -plane as shown in the figure below, is:



- (a) $\frac{5}{e} + e$ (b) $e - \frac{5}{e}$ (c) $\frac{5}{e} - e$ (d) $-\frac{5}{e} - e$
25. The residue of e^{2/z^2} at $z = 0$ is:
 (a) 0 (b) ∞ (c) $2\pi i$ (d) 2



26. The value of the integral $\oint_C \frac{z^3}{(z-1)^2} dz$, where C is the circle $|z|=4$ is
 (a) $6\pi i$ (b) $-6\pi i$ (c) $2\pi i$ (d) Zero
27. The leading term in Laurent expansion of $f(z) = \frac{1}{z(1-z)^2}$ around $z=0$ is
 (a) $\frac{1}{z^3}$ (b) $\frac{1}{z^2}$ (c) $\frac{1}{z}$ (d) 1
28. The value of $\oint_C \frac{e^z(z+3)}{z-2} dz$ where C is a unit circle with centre at origin is:
 (a) $5e^2$ (b) $\frac{5}{2\pi i} e^2$ (c) 0 (d) $10\pi i e^2$
29. If $f(z) = \frac{Z^2 + a^2 - 2aZ}{(Z-a)^3(Z-b)}$, then $Z=a$ is:
 (a) Pole of order 1 (b) Pole of order 2 (c) Pole of order 3 (d) An essential singularity.
30. Which one of the following functions is not analytic?
 (a) $f(z) = |z|$ (b) $f(z) = z^2$ (c) $f(z) = \cos z$ (d) $f(z) = \sin z$
31. The value of the integral $\oint_C \frac{\cos \pi z}{z(z+1)}$ where C is the circle $|z-1|=3$ is:
 (a) 0 (b) πi (c) $2\pi i$ (d) $4\pi i$
32. What is the type of singularity possessed by the function, $f(Z) = \exp\left[\frac{1}{(Z-1)^2}\right]$ at $Z=1$?
 (a) Simple pole (b) Branch point
 (c) Pole of second order (d) Essential singularity
33. The residue of the function $f(z) = \frac{1}{z(z+2)^2}$ at $z=-2$ is:
 (a) $-\frac{1}{4}$ (b) $-\frac{1}{2}$ (c) $-\frac{\pi i}{2}$ (d) $4\pi i$



Common data for Q. 34 and Q. 35.

Consider a function $f(z) = \frac{z \sin z}{(z - \pi)^2}$ of a complex variable z .

34. Which of the following statements is TRUE for the function $f(z)$?
- (a) $f(z)$ is analytic everywhere in the complex plane
 (b) $f(z)$ has a zero at $z = \pi$
 (c) $f(z)$ has a pole of order 2 at $z = \pi$ (d) $f(z)$ has a simple pole at $z = \pi$.
35. Consider a counterclockwise circular contour $|z| = 1$ about the origin. The integral $\oint_C f(z) dz$ over this contour is:
- (a) $-i\pi$ (b) Zero (c) $i\pi$ (d) $2i\pi$
36. The value of the contour integral, $\frac{1}{2\pi i} \oint_C dz f(z)$. Where $f(z) = \frac{z}{2} + \frac{1}{z} + \frac{2z}{z^2 - 1}$ and the contour C is a circle of radius 2 centered at the origin, traversed in the counter-clockwise direction is:
- (a) 2 (b) 1/2 (c) 1 (d) 3
37. Which of the following is an analytic function of the complex variable $z = x + iy$ in the domain $|z| < 2$?
- (a) $(3 + x - iy)^7$ (b) $(1 + x + iy)^4 (7 - x - iy)^3$
 (c) $(1 - 2x - iy)^4 (3 - x - iy)^3$ (d) $(x + iy - 1)^{1/2}$
38. The first few terms in the Taylor series expansion of the function $f(x) = \sin x$ around $x = \frac{\pi}{4}$ are
- (a) $\frac{1}{\sqrt{2}} \left[1 + \left(x - \frac{\pi}{4}\right) + \frac{1}{2!} \left(x - \frac{\pi}{4}\right)^2 + \frac{1}{3!} \left(x - \frac{\pi}{4}\right)^3 \dots \right]$
 (b) $\frac{1}{\sqrt{2}} \left[1 + \left(x - \frac{\pi}{4}\right) - \frac{1}{2!} \left(x - \frac{\pi}{4}\right)^2 - \frac{1}{3!} \left(x - \frac{\pi}{4}\right)^3 \dots \right]$
 (c) $\left[\left(x - \frac{\pi}{4}\right) - \frac{1}{3!} \left(x - \frac{\pi}{4}\right)^3 \dots \right]$ (d) $\frac{1}{\sqrt{2}} \left[1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} \dots \right]$
39. The residue of the function $f(z) = \frac{1}{1 + z^2}$ at $z = i$ is
- (a) $-i$ (b) $2i$ (c) $\frac{1}{2i}$ (d) 1
40. The value of the integral $\oint_C e^{yz} dz$, using the contour C of circle with unit radius $|z| = 1$ is:
- (a) 0 (b) $1 - 2\pi i$ (c) $1 + 2\pi i$ (d) $2\pi i$



41. The value of the integral $\int_0^{\infty} \frac{\ln x}{(x^2+1)^2} dx$ is:
 (a) 0 (b) $-\pi/4$ (c) $-\pi/2$ (d) $\pi/2$
42. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1 \leq |z| \leq 2$, and around $z=1$ is
 (a) $\frac{1}{2} \left[1+z+z^2+z^3+\dots \right] \left[1+\frac{z}{2}+\frac{z^2}{4}+\frac{z^3}{8}+\dots \right]$
 (b) $\frac{1}{1-z} - z - (1-z)^2 + (1-z)^3 + \dots$ (c) $\frac{1}{z^2} \left[1+\frac{1}{z}+\frac{1}{z^2}+\dots \right] \left[1+\frac{2}{z}+\frac{4}{z^2}+\dots \right]$
 (d) $2(z-1)+5(z-1)^2+7(z-1)^3+\dots$
43. Let $u(x, y) = x + \frac{1}{2}(x^2 - y^2)$ be the real part of an analytic function $f(z)$ of the complex variable $z = x + iy$. the imaginary part of $f(z)$ is
 (a) $y + xy$ (b) xy (c) y (d) $y^2 - x^2$
44. The value of the integral $\int_{-\infty}^{\infty} \frac{1}{t^2 - R^2} \cos\left(\frac{\pi t}{2R}\right) dt$ is
 (a) $-2\pi/R$ (b) $-\pi/R$ (c) π/R (d) $2\pi/R$
45. If $z = x + iy$ then the function,

$$f(x, y) = (1+x+y)(1+x-y) + a(x^2 - y^2) - 1 + 2iy(1-x-ax)$$
 where 'a' is a real parameter, is analytic in the complex 'z' plane if $a=0$
 (a) 1 (b) +1 (c) 0 (d) i
46. The Taylor expansion of the function $\ln(\cosh x)$, where 'x' is real, about the point $x=0$ starts with the following terms:
 (a) $-\frac{1}{2}x^2 + \frac{1}{12}x^4 + \dots$ (b) $\frac{1}{2}x^2 - \frac{1}{12}x^4 + \dots$
 (c) $-\frac{1}{2}x^2 + \frac{1}{6}x^4 + \dots$ (d) $\frac{1}{2}x^2 + \frac{1}{6}x^4 + \dots$
47. The value of the integral $\int_C \frac{z^3 dz}{z^2 - 5z + 6}$, where C is a closed contour defined by the equation $2|z| - 5 = 0$, traversed in the anti-clockwise direction, is:
 (a) $-16\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) $2\pi i$
48. Compute $\lim_{z \rightarrow 0} \frac{\operatorname{Re}(z^2) + \operatorname{Im}(z^2)}{z^2}$
 (a) The limit does not exist (b) 1
 (c) $-i$ (d) -1



49. Which of the following functions cannot be the real part of a complex analytic function of $z = x + iy$?

- (a) x^2y (b) $x^2 - y^2$ (c) $x^3 - 3xy^2$ (d) $3x^2y - y - y^3$

50. The integral $\int_0^{\infty} \frac{dx}{4 + x^4}$

evaluates to

- (a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$

51. Given that the integral $\int_0^{\infty} \frac{dx}{y^2 + x^2} = \frac{\pi}{2y}$, the value of $\int_0^{\infty} \frac{dx}{(y^2 + x^2)^2}$ is

- (a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$ (c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

52. The partial differential equation, $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$

upon change of variables to $z = x + iy$, $z^* = x - iy$ becomes

- (a) $\frac{\partial u}{\partial z} + \frac{\partial u}{\partial z^*} = 0$ (b) $z \frac{\partial u}{\partial z} + z^* \frac{\partial u}{\partial z} = 0$ (c) $z \frac{\partial u}{\partial z} + z^* \frac{\partial u}{\partial z^*} = 0$ (d) $z \frac{\partial u}{\partial z} - z^* \frac{\partial u}{\partial z^*} = 0$

53. If C is the contour defined by $|z| = \frac{1}{2}$, the value of the integral

$$\oint_C \frac{dz}{\sin^2 z}$$

is

- (a) ∞ (b) $2\pi i$ (c) 0 (d) πi

54. The function $\Phi(x, y, z, t) = \cos(z - vt) + \operatorname{Re}(\sin(x + iy))$ satisfies the equation

- (a) $\frac{1}{v^2} \frac{\partial^2 \Phi}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi$ (b) $\left(\frac{1}{v^2} \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2} \right) \Phi$
 (c) $\left(\frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} \right) \Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) \Phi$ (d) $\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right) \Phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) \Phi$



Complex Analysis Answer Key

1. (c)	2. (b)	3. (c)	4. (b)	5. (a)	6. (d)	7. (c)
8. (d)	9. (b)	10. (a)	11. (b)	12. (a)	13. (c)	14. (a,c)
15. (a)	16. (a)	17. (d)	18. (a)	19. (b)	20. (d)	21. (d)
22. (c)	23. (b)	24. (c)	25. (a)	26. (a)	27. (c)	28. (c)
29. (a)	30. (a)	31. (d)	32. (d)	33. (a)	34. (c)	35. (b)
36. (d)	37. (b)	38. (b)	39. (c)	40. (d)	41. (b)	42. (b)
43. (a)	44. (a)	45. (a)	46. (b)	47. (a)	48. (a)	49. (a)
50. (d)	51. (b)	52. (d)	53. (c)	54. (a)		



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CSIR-UGC-NET/JRF | GATE Physics
MATHEMATICAL PHYSICS

Assignment-IV: Differential Equations

- Solution of $\frac{d^2y}{dx^2} - \frac{5dy}{dx} + 6 = 0$ is
(a) $C_1 e^{-3x} + C_2 e^{-2x}$ (b) $C_1 \cos x + C_2 \sin x$
(c) $C_1 \cosh x + C_2 \sinh x$ (d) $C_1 e^{3x} + C_2 e^{2x}$
- Solution of $\frac{d^2x}{dt^2} + \frac{2dx}{dt} + 1 = 0$ is
(a) $x = C_1 e^{-1} + C_2 e^{-1}$ (b) $x = C_1 e^1 + C_2 e^1$ (c) $x = C_1 e^{-1} + C_2 t e^{-1}$ (d) none
- The value of $P_3(x)$ is
(a) $\frac{1}{2}(3x^2 - 1)$ (b) $3x$ (c) $\frac{1}{2}(5x^3 - 3x)$ (d) $\frac{1}{2}(5x^3 - 1)$
- If $f(x) = x^4 - 2 = \sum_{n=0}^{\infty} a_n P_n(x)$, then a_3 is
(a) $\frac{1}{2}$ (b) 1 (c) -1 (d) 0
- The value of $\frac{J_{1/2}(x)}{J_{-1/2}(x)}$ is:
(a) 0 (b) $\tan x$ (c) $\cot x$ (d) $\tan hx$
- The points, where the series solution of the Legendre differential equation $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \frac{3}{2}\left(\frac{3}{2}+1\right)y = 0$ will diverge, are located at:
(a) 0 and 1 (b) 0 and -1 (c) -1 and 1 (d) $\frac{3}{2}$ and $\frac{5}{2}$

Statement for Linked Answer Q. 7 and Q. 8:

For the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

- One of the solutions is:
(a) e^x (b) $\ln x$ (c) e^{-x^2} (d) e^{x^2}



8. The second linearly independent solution is:
 (a) e^{-x} (b) xe^x (c) x^2e^x (d) x^2e^{-x}
9. Consider the differential equation $d^2x/dt^2 + 2 dx/dt + x = 0$. At time $t = 0$, it is given that $x = 1$ and $dx/dt = 0$. At $t = 1$, the value of x is given by:
 (a) $1/e$ (b) $2/e$ (c) 1 (d) $3/e$
10. The solution of the differential equation,

$$(1+x) \frac{d^2y(x)}{dx^2} + x \frac{dy(x)}{dx} - y(x) = 0 \text{ is:}$$

- (a) $Ax^2 + B$ (b) $Ax + Be^{-x}$ (c) $Ax + Be^x$ (d) $Ax + Bx^2$
 where A and B are constants.

11. The solution of the system of differential equations,

$$\frac{dy}{dx} = y - z \text{ and } \frac{dz}{dx} = -4y + z$$

is given by (for A and B are arbitrary constants)

- (a) $y(x) = Ae^{3x} + Be^{-x}; z(x) = -2Ae^{3x} + 2Be^{-x}$ (b) $y(x) = Ae^{3x} + Be^{-x}; z(x) = 2Ae^{3x} + 2Be^{-x}$
 (c) $y(x) = Ae^{3x} + Be^{-x}; z(x) = 2Ae^{3x} - 2Be^{-x}$ (d) $y(x) = Ae^{3x} + Be^{-x}; z(x) = -2Ae^{3x} - 2Be^{-x}$
12. Legendre's differential equation: $(1-x^2) d^2y/dx^2 - 2x dy/dx + n(n+1)y = 0$ has an ordinary point at $x =$
 (a) -1 (b) 0 (c) 1 (d) ∞
13. For the differential equation

$$z^2 \frac{d^2y}{dz^2} + z \frac{dy}{dz} + y = 0$$

- (a) $z = \infty$ is an ordinary point
 (b) $z = 0$ is a regular singular and $z = \infty$ is an irregular singular point
 (c) $z = 0$ and $z = \infty$ both are regular singular points
 (d) $z = 0$ is an irregular singular point
14. The Legendre polynomial $P_n(x)$, where $-1 \leq x \leq +1$
 (a) Is singular at $x = \pm 1$ (b) Satisfies $\int_{-1}^1 dx P_n^2(x) = 1$
 (c) Satisfies $\int_{-1}^1 dx P_n(x) = 0$ for $n \geq 1$ (d) Is always an even function of x .

15. For the Bessel equation, $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0$, where n is an integer, the maximum number of linearly independent solutions, well-defined at $x = 0$ is
 (a) zero (b) one (c) two (d) three



16. The general solution of the differential equation, $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = 0$ is:
 (a) $C_1 e^{2x} + C_2 e^{-2x}$ (b) $C_1 e^{2x} + C_2 x e^{2x}$ (c) $C_1 e^{-2x} + i C_2 e^{2x}$ (d) $C_1 e^{-2x} + C_2 x e^{-2x}$
 Here C_1 and C_2 are arbitrary constants.
17. In order to obtain the solution of the initial value problem of the equation of motion, $\frac{d^2 x}{dt^2} + 5 \frac{dx}{dt} + 16x = 0$.
 How many initial conditions are required?
 (a) 0 (b) 1 (c) 2 (d) 3
18. The general solution of the linear differential equation, $\frac{d^4 u}{dx^4} = 0$ is, $u(x)$ equal to
 (a) 0 (b) $c_3 t + c_4$ (c) $c_2 t^2 + c_3 t + c_4$ (d) $c_1 t^3 + c_2 t^2 + c_3 t + c_4$
19. The regular singular points of the associated Legendre differential equation in the finite domain are
 (a) 1, -1 (b) 0 (c) ∞ (d) 2
20. The Wronskian of the linearly independent solutions of the differential equation $\frac{d^2 y}{dx^2} + 4y = 0$ is:
 (a) Constant (b) 0 (c) ∞ (d) Undetermined
21. The solutions to the differential equation $\frac{dy}{dx} = -\frac{x}{y+1}$ are a family of
 (a) Circles with different radii (b) Circles with different centres.
 (c) Straight lines with different slopes (d) Straight lines with different intercepts on the y-axis
22. Given the Legendre polynomials $P_0(x) = 1$, $P_1(x) = x$, and $P_2(x) = \frac{3x^2 - 1}{2}$, then the polynomial $(3x^2 + x - 1)$ can be expressed as
 (a) $P_2(x) - P_1(x)$ (b) $2P_2(x) + P_1(x)$
 (c) $P_2(x) + P_1(x)$ (d) $2P_2(x) + P_1(x) + P_0(x)$
23. The generating function $F(x, t) \equiv \sum_{n=0}^{\infty} P_n(x) t^n$ for the Legendre polynomials $P_n(x)$ is
 $F(x, t) = (1 - 2xt + t^2)^{-1/2}$. The value of $P_3(-1)$ is:
 (a) 5/2 (b) 3/2 (c) +1 (d) -1



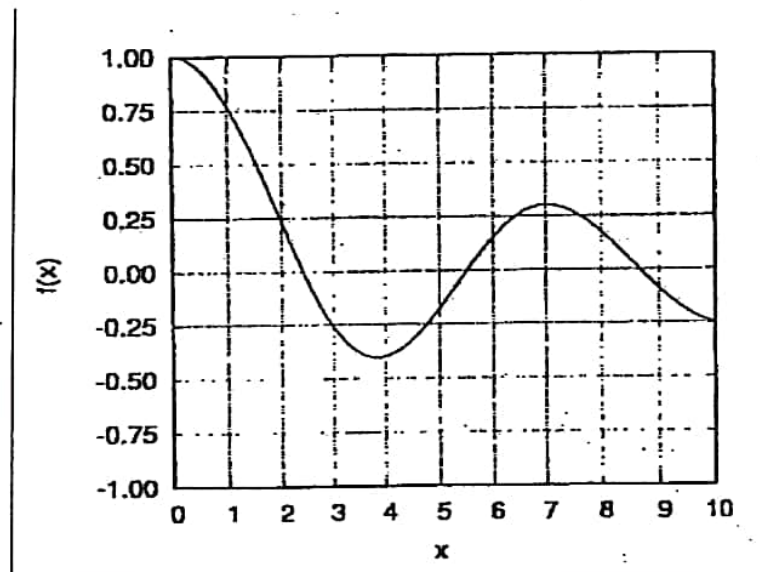
24. Let $x_1(t)$ and $x_2(t)$ be two linearly independent solutions of the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + f(t)x = 0$, and let $w(t) = x_1(t)\frac{dx_2(t)}{dt} - x_2(t)\frac{dx_1(t)}{dt}$ if $w(0) = 1$, then $w(1)$ is given by
 (a) 1 (b) e^2 (c) $1/e$ (d) $1/e^2$
25. If one expands $f(x) = x^4 - 2$ around $x = 0$ in terms of the Legendre Polynomials $\left(f(x) = \sum_{n=0}^{\infty} a_n P_n(x)\right)$, the coefficient a_3 is:
 (a) 1 (b) 0 (c) 3 (d) e^2
26. The most general solution of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6 = 0$ in terms of arbitrary constants C_1 and C_2 is
 (a) $C_1 e^{-3x} + C_2 e^{-2x}$ (b) $C_1 \sin 3x + C_2 \cos 2x$ (c) $C_1 e^{3x} + C_2 e^{2x}$ (d) $C_1 e^{-3ix} + C_2 e^{2ix}$
27. The solution of the differential equation $x \frac{dy}{dx} + y = 1$, given the condition $y = 0$ at $x = 1$ is
 (a) $y = (1 - x^2) \ln x$ (b) $y = \frac{x^2 - 1}{x} e^{-x}$ (c) $y = \frac{x - 1}{x}$ (d) $y = \sinh(x)$
28. Let $y(x)$ be a continuous real function in the range 0 and 2π , satisfying the inhomogeneous differential equation: $\sin x \frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} = \delta\left(x - \frac{\pi}{2}\right)$. The value of dy/dx at the point $x = \pi/2$
 (a) is continuous (b) has a discontinuity of 3
 (c) has a discontinuity of $1/3$ (d) has a discontinuity of 1
29. The differential equation,

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

 has the complete solution, in terms of arbitrary constants A and B,
 (a) $A \exp x + B x \exp x$ (b) $A \exp x + B \exp(-x)$
 (c) $A \exp x + B x \exp(-x)$ (d) $x \{A \exp x + B \exp(-x)\}$



30. The graph of the function $f(x)$ as shown below is best described by



- (a) The bessel function $J_0(x)$ (b) $\cos x$
 (c) $e^{-x} \cos x$ (d) $\frac{1}{x} \cos x$
31. A function $f(x)$ obeys the differential equation $\frac{d^2 f}{dx^2} - (3-4i)f = 0$ and satisfies the conditions $f(0) = 1$ and $f(x) \rightarrow 0$ as $x \rightarrow \infty$. The value of $f(\pi)$ is:
 (a) $e^{2\pi}$ (b) $e^{-2\pi}$ (c) $-e^{-2\pi}$ (d) $-e^{2\pi i}$
32. Consider the differential equation $\frac{dG(x)}{dx} + kG(x) = \delta(x)$, where k is a constant. Which of the following statements is true?
 (a) Both $G(x)$ and $G'(x)$ are continuous at $x = 0$.
 (b) $G(x)$ is continuous at $x = 0$ but $G'(x)$ is not.
 (c) $G(x)$ is discontinuous at $x = 0$.
 (d) The continuity properties of $G(x)$ and $G'(x)$ at $x = 0$ depend on the value of k .
33. Given,
 $\sum_{n=0}^{\infty} P_n(x) t^n = (1 - 2xt + t^2)^{-1/2}$, for $|t| < 1$, the value of $P_5(-1)$ is
 (a) 0.26 (b) 1 (c) 0.5 (d) -1
34. Consider the differential equation

$$\frac{d^2 x}{dt^2} + 2 \frac{dx}{dt} + x = 0$$
 with the initial conditions $x(0) = 0$ and $\dot{x}(0) = 1$. The solution $x(t)$ attains its maximum value when 't' is
 (a) 1/2 (b) 1 (c) 2 (d) ∞



Differential Equations Answer Key

1. (d)	2. (c)	3. (c)	4. (d)	5. (b)	6. (d)	7. (a)
8. (b)	9. (b)	10. (b)	11. (a)	12. (b)	13. (a)	14. (c)
15. (c)	16. (b)	17. (c)	18. (d)	19. (a)	20. (a)	21. (a)
22. (b)	23. (d)	24. (d)	25. (b)	26. (c)	27. (c)	28. (d)
29. (a)	30. (a)	31. (c)	32. (c)	33. (d)	34. (b)	





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MATHEMATICAL PHYSICS

Assignment V: Fourier Series, Fourier Transform & Laplace Transform

- Inverse Laplace transform of $\frac{a}{s^2 + a^2}$ is
(a) $\sin at$ (b) $\sinh at$ (c) $\cos at$ (d) $\cosh at$
- Using the fourier series of $f(x) = x^2, -\pi < x < \pi$. The value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is
(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{6}$ (c) $\frac{\pi^2}{12}$ (d) π^2
- Fourier transform of Gaussian function e^{-x^2} is:
(a) $\frac{1}{\sqrt{2}} e^{-\omega^2/4}$ (b) $\frac{1}{\sqrt{2\pi}} e^{-\omega^2/4}$ (c) $e^{-\omega^2}$ (d) $e^{-\omega^2/2}$
- Inverse Laplace transform of $\frac{s+1}{s^2-4}$ is:
(a) $\cos 2x + \frac{1}{2} \sin 2x$ (b) $\cos x + \frac{1}{2} \sin x$
(c) $\cosh x + \frac{1}{2} \sinh x$ (d) $\cosh 2x + \frac{1}{2} \sinh 2x$
- The k th Fourier component of $f(x) = \delta(x)$ is:
(a) 1 (b) 0 (c) $(2\pi)^{-1/2}$ (d) $(2\pi)^{-3/2}$
- If $\bar{f}(s)$ is the Laplace transform of $f(t)$ the Laplace transform of $f(at)$, where a is a constant, is
(a) $\frac{1}{a} \bar{f}(s)$ (b) $\frac{1}{a} \bar{f}(s/a)$ (c) $\bar{f}(s)$ (d) $\bar{f}(s/a)$
- The Laplace transform of $f(t) = \sin \pi t$ is $F(s) = \frac{\pi}{(s^2 + \pi^2)}, s > 0$. Therefore, the Laplace transform of $t \sin \pi t$ is:
(a) $\frac{\pi}{s^2(s^2 + \pi^2)}$ (b) $\frac{2\pi}{s^2(s^2 + \pi^2)^2}$ (c) $\frac{2\pi s}{(s^2 + \pi^2)^2}$ (d) $\frac{2\pi}{(s^2 + \pi^2)^2}$



8. A periodic function $f(x) = x$ for $-\pi < x < +\pi$ has the Fourier series representation $f(x) = \sum_{n=1}^{\infty} \left(-\frac{2}{n}\right) (-1)^n \sin nx$. Using this, one finds the sum $\sum_{n=1}^{\infty} n^{-2}$ to be:
- (a) $2 \ln 2$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{6}$ (d) $\pi \ln 2$
9. The Fourier transform $F(k)$ of a function $f(x)$ is defined as:
 $F(k) = \int_{-\infty}^{\infty} dx f(x) \exp(ikx)$. Then $F(k)$ for $f(x) = \exp(-x^2)$ is:
 [Given: $\int_{-\infty}^{\infty} \exp(-x^2) dx = \sqrt{\pi}$]
- (a) $\pi \exp(-k)$ (b) $\sqrt{\pi} \exp\left(\frac{-k^2}{4}\right)$ (c) $\frac{\sqrt{\pi}}{2} \exp\left(\frac{-k^2}{2}\right)$ (d) $\sqrt{2\pi} \exp(-k^2)$
10. The Fourier transform of the function $f(x)$ is $F(k) = \int e^{ikx} f(x) dx$. The Fourier transform of $df(x)/dx$ is:
- (a) $dF(k)/dk$ (b) $\int F(k)/dk$ (c) $-ikF(k)$ (d) $ikF(k)$
11. Fourier transform of which of the following functions does not exist?
- (a) $e^{-|x|}$ (b) xe^{-x^2} (c) e^{x^2} (d) e^{-x^2}
12. If $f(x)$ be an even, period function of x which is continuous. Then
- (a) It can be represented by a cosine series. (b) It can be represented by a sine series
 (c) It cannot be represented by a Fourier series
 (d) It needs both sine and cosine terms in its Fourier expansion
13. Laplace transform $L\{F(t)\}$ of a function $F(t)$ is $f(s)$. Which of the following expression is FALSE?
- (a) $\frac{d^n}{ds^n} f(s) = (-1)^n L\{t^n F(t)\}$ (b) $f(s+a) = L\{e^{-at} F(t)\}$
 (c) $L\left\{\sum_{i=1}^n a_i F_i(t)\right\} = \sum_{i=1}^n a_i L\{F_i(t)\}$ (d) $L\{F(at)\} = \frac{1}{a} f(s)$
14. Which of the following is NOT a periodic function of θ ?
- (a) $\sin \theta + \pi \cos \theta$ (b) $\sin(\pi\theta) + \cos(\pi\theta)$
 (c) $\sin \theta + \cos(\pi\theta)$ (d) $\sin \theta + \cos(\theta + \pi)$
15. The Fourier series expansion of the function $f(x) = x(\pi - x)$, $0 \leq x \leq \pi$, is:
- $\frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \dots\right)$. The value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is deduced as
- (a) $\frac{\pi^2}{6}$ (b) $\frac{2\pi^2}{6}$ (c) $-\frac{\pi^2}{6}$ (d) Zero



16. The Fourier coefficients a_n in the Fourier series expansion.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx, \text{ for } f(x) = \begin{cases} -1, & -\pi \leq x \leq 0 \\ +1, & 0 \leq x \leq \pi \end{cases}, \text{ are}$$

(a) $\frac{2}{n\pi} [1 - (-1)^n]$ (b) $\frac{4}{n\pi}$ (c) $\frac{2}{n\pi} [1 + (-1)^n]$ (d) 0

17. Coefficient of $\cos x$ in the Fourier series expansion of x over 0 to 2π is:

(a) 0 (b) 1 (c) 2 (d) $1/2$

18. The function $f(x) = x^2$, $0 < x < 2$ has a Fourier series representation

$$x^2 = \frac{4\pi^2}{3} + \sum_{n=1}^{\infty} \left(\frac{4}{n^2} \cos nx - \frac{4\pi}{n} \sin nx \right)$$

At $x=0$ and 2π , the series converges to $2\pi^2$. Then the value of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is:

(a) $2\pi^2$ (b) π^2 (c) $\frac{\pi^2}{3}$ (d) $\frac{\pi^2}{6}$

19. Consider the Fourier series corresponding to the function

$$f(x) = \begin{cases} 3, & 0 < x < 5 \\ -3, & -5 < x < 0 \end{cases}$$

Then the Fourier series contains.

- (a) Cosine terms only (b) Both cosine and sine terms
(c) Sine terms only (d) Sine terms and non-zero constant terms.

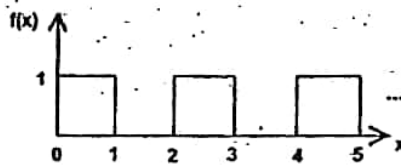
20. The constant coefficient in the Fourier series of the function

$$f(x) = \begin{cases} 0 & -5 < x < 0 \\ 3 & 0 < x < 5 \end{cases} \text{ is:}$$

(a) 3 (b) $\frac{3}{2}$ (c) 0 (d) 5

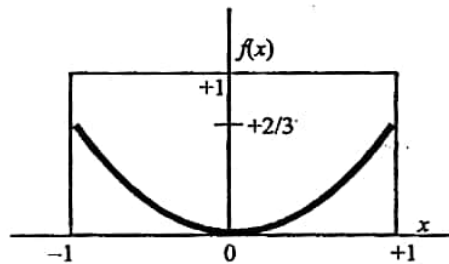


21. The graph of the function $f(x) = \begin{cases} 1 & \text{for } 2n \leq x \leq 2n+1 \\ 0 & \text{for } 2n+1 \leq x \leq 2n+2 \end{cases}$ (where $n=0, 1, 2, \dots$) is shown below.



Its Laplace transform $\bar{f}(s)$ is

- (a) $\frac{1+e^{-s}}{s}$ (b) $\frac{1-e^{-s}}{s}$ (c) $\frac{1}{s(1+e^{-s})}$ (d) $\frac{1}{s(1-e^{-s})}$
22. Fourier transform of the derivative of the Dirac δ -function, namely $\delta'(x)$, is proportional to
 (a) 0 (b) 1 (c) sink (d) ik
23. A student is asked to find a series approximation for the function $f(x)$ in the domain $-1 \leq x \leq +1$, as indicated by the thick line in the figure below.



The student represents the function by a sum of three terms

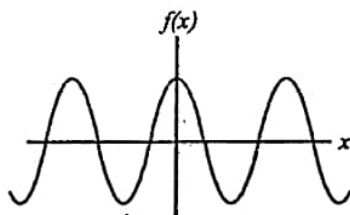
$$f(x) \approx a_0 + a_1 \cos \frac{\pi x}{2} + a_2 \sin \frac{\pi x}{2}$$

Which of the following would be the best choices for the coefficients a_0 , a_1 and a_2 ?

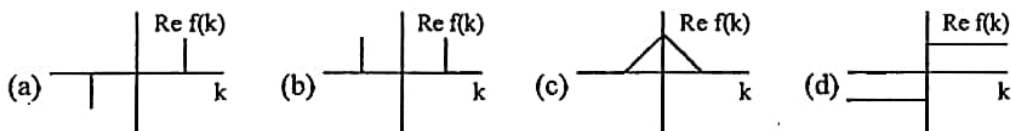
- (a) $a_0 = 1, a_1 = -\frac{1}{3}, a_2 = 0$ (b) $a_0 = \frac{2}{3}, a_1 = -\frac{2}{3}, a_2 = 0$
- (c) $a_0 = \frac{2}{3}, a_1 = 0, a_2 = -\frac{2}{3}$ (d) $a_0 = -\frac{1}{3}, a_1 = 0, a_2 = -1$



24. The graph of a real periodic function $f(x)$ for the range $[-\infty, \infty]$ is shown below



Which of the following graphs represents the real part of its Fourier transform?



Fourier Series, Fourier Transform & Laplace Transform_Answer Key

- | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (a) | 2. (b) | 3. (a) | 4. (d) | 5. (c) | 6. (b) | 7. (c) |
| 8. (c) | 9. (b) | 10. (c) | 11. (c) | 12. (a) | 13. (d) | 14. (c) |
| 15. (a) | 16. (d) | 17. (a) | 18. (d) | 19. (c) | 20. (b) | 21. (c) |
| 22. (b) | 23. (b) | 24. (b) | | | | |





CSIR-UGC-NET/JRF | GATE Physics

Assignment VI: Probability

- The probability of getting m heads out of n trials of a coin is
 (a) $\frac{m!}{n!(n-m)!} \left(\frac{1}{2}\right)^m$ (b) $\frac{n!}{m!(n-m)!} \left(\frac{1}{2}\right)^n$ (c) $\frac{n!}{m!(n-m)!} \left(\frac{1}{2}\right)$ (d) $\frac{m!}{n!(n-m)!} \left(\frac{1}{2}\right)^n$
- There are 3 cows and 4 bulls in a field, the probability of taking birth of a calf by the combination of a cow and a bull is:
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{12}$
- The probability of throwing more than 4 by one dice in a throw is:
 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$
- An unbiased dice is thrown three times successively. The probability that the numbers of dots on the uppermost surface add up to 16 is
 (a) $\frac{1}{16}$ (b) $\frac{1}{36}$ (c) $\frac{1}{108}$ (d) $\frac{1}{216}$
- An unbiased coin is tossed n times. The probability that exactly m heads will come up is
 (a) $\frac{1}{2^n} \frac{n!}{m!(n-m)!}$ (b) $\frac{1}{2^n} \frac{n!}{m!(n-m)!}$ (c) $\frac{n}{2^m}$ (d) $\frac{m}{2^n}$
- If a three digit number is chosen at random, the probability for the middle digit to be 3 is
 (a) $1/10$ (b) $(1/10)^3$ (c) $1/3$ (d) $1/30$
- Suppose a team is equally likely to win or lose a match. In a series of two matches, if we know that it has won at least one match, what is the probability that it has won both matches?
 (a) $3/4$ (b) $1/2$ (c) $1/3$ (d) $1/4$
- A coin and a die are thrown independently. The probability that a 'head' appears on the coin and an even integer on the die (assuming both are unbiased) is:
 (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{12}$
- An unbiased die is cast twice. The probability that the positive different (bigger - smaller) between the two numbers is 2 is
 (a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$



10. A ball is picked at random from one of two boxes that contain 2 black and 3 black and 4 white balls respectively. What is the probability that it is white?
(a) $34/70$ (b) $41/70$ (c) $36/70$ (d) $29/70$
11. In a series of five cricket matches, one of the captains calls "Heads" every time when the toss is taken. The probability that he will win 3 times and lose 2 times is
(a) $1/8$ (b) $5/8$ (c) $3/16$ (d) $5/16$
12. There are on average 20 buses per hour at a point, but at random times. The probability that there are no buses in five minutes is closest to
(a) 0.07 (b) 0.60 (c) 0.36 (d) 0.19
13. Two drunks start out together at the origin, each having equal probability of making a step simultaneously to the left or right along the x axis. The probability that they meet after n steps is
(a) $\frac{1}{4^n} \frac{2n!}{n!^2}$ (b) $\frac{1}{2^n} \frac{2n!}{n!^2}$ (c) $\frac{1}{2^n} 2n!$ (d) $\frac{1}{4^n} n!$
14. A box contains 100 coins out of which 99 are fair coins and 1 is a double-headed coin. Suppose you choose a coin at random and toss it 3 times. It turns out that the results of all 3 tosses are heads. What is the probability that the coin you have drawn is the double-headed one?
(a) 0.99 (b) 0.925 (c) 0.075 (d) 0.01
15. A loaded dice has the probabilities $\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}$ and $\frac{6}{21}$ of turning up 1, 2, 3, 4, 5 and 6, respectively. If it is thrown twice, what is the probability that the sum of the numbers that turn up is even?
(a) $\frac{144}{441}$ (b) $\frac{225}{441}$ (c) $\frac{221}{441}$ (d) $\frac{220}{441}$
16. A child makes a random walk on a square lattice of lattice constant a taking a step in the north, east, south, or west directions with probabilities 0.255, 0.255, 0.245, and 0.245, respectively. After a large number of steps, N , the expected position of the child with respect to the starting point is at a distance
(a) $\sqrt{2} \times 10^{-2} Na$ in the north-east direction (b) $\sqrt{2N} \times 10^{-2} a$ in the north-east direction
(c) $2\sqrt{2} \times 10^{-2} Na$ in the south-east direction (d) 0
17. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps?
(a) $3/8$ (b) $5/16$ (c) $1/4$ (d) $1/16$

Probability Answer Key

- | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (b) | 2. (d) | 3. (b) | 4. (b) | 5. (b) | 6. (a) | 7. (d) |
| 8. (a) | 9. (b) | 10. (b) | 11. (d) | 12. (d) | 13. (a) | 14. (c) |
| 15. (b) | 16. (a) | 17. (a) | | | | |





CSIR-UGC-NET/JRF | GATE Physics

MATHEMATICAL PHYSICS

Assignment VII: Miscellaneous Problems

(Tensors, Numerical Techniques, Group Theory, Beta-Gamma Function, Series, Dirac-Delta Function)

- Given that $T_{ij} = \frac{1}{2}(x_i y_j - x_j y_i)$ and $S_{ij} = \frac{1}{2}(x_i y_j + x_j y_i)$. Then $T_{ij} S_{ij}$ is:
 (a) 0 (b) $\sum x_i^4 - y_i^4$ (c) $\sum x_i^2 y_i^2$ (d) $\sum (x_i^2 - y_i^2)$
- The number of independent components of symmetric tensor of rank 2 in n-dimensional space is:
 (a) 2n (b) n^2 (c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2}$
- The number of independent components of antisymmetric tensor of rank 2 in n-dimensional space is:
 (a) n (b) n^2 (c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2}$
- $\int_0^\infty x e^{-x^2} dx$ is
 (a) $\frac{1}{2}$ (b) $\sqrt{\frac{\pi}{2}}$ (c) $\sqrt{\frac{2}{\pi}}$ (d) 1
- The integral formula $\int_{x_0}^{x_0+nh} y dx = \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$
 (a) Simpson's $\frac{3}{8}$ rule (b) Simpson's $\frac{1}{3}$ rule (c) Trapezoidal rule (d) Weddle's rule.
- The average value of the function $f(x) = 4x^3$ in the interval 1 to 3 is:
 (a) 15 (b) 20 (c) 40 (d) 80
- The average of the function $f(x) = \sin x$ in the interval $(0, \pi)$ is:
 (a) $\frac{1}{2}$ (b) $\frac{2}{\pi}$ (c) $\frac{1}{\pi}$ (d) $\frac{4}{\pi}$
- S_{ij} and A_{ij} represent a symmetric and an antisymmetric real-valued tensor respectively in three dimensions. The number of independent components of S_{ij} and A_{ij} are:
 (a) 3 and 6 respectively (b) 6 and 3 respectively
 (c) 6 and 6 respectively (d) 9 and 6 respectively.
- Consider the four statements given below about the function $f(x) = x^4 - x^2$ in the range $-\infty < x < +\infty$. Which one of the following statements is correct?
 P: the plot of $f(x)$ versus x has two maxima and two minima
 Q: the plot of $f(x)$ versus x cuts the x axis at four points
 R: the plot of $f(x)$ versus x has three extrema



S: no part of the plot $f(x)$ versus x lies in the fourth quadrant

Pick the right combination of correct choices from those given below.

- (a) P and R (b) R only (c) R and S (d) P and Q

10. If $u(x, y, z, t) = f(x + i\beta y - vt) + g(x - i\beta y - vt)$, where f and g are arbitrary and twice differentiable functions,

is a solution of the wave equation, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$, then β is

- (a) $\left(1 - \frac{v}{c}\right)^{1/2}$ (b) $\left(1 - \frac{v}{c}\right)$ (c) $\left(1 - \frac{v^2}{c^2}\right)^{1/2}$ (d) $\left(1 - \frac{v^2}{c^2}\right)$

11. The Dirac delta function $\delta(x)$ can be represented by:

- (a) $\frac{1}{\pi} \lim_{N \rightarrow \infty} \frac{\sin(Nx)}{x}$ (b) $\frac{1}{\pi} \lim_{\epsilon \rightarrow 0^+} \frac{\epsilon}{x^2 + \epsilon^2}$
 (c) $\frac{1}{\sqrt{x}} \lim_{\epsilon \rightarrow +\infty} \frac{1}{\sqrt{\epsilon}} \exp(-x^2/\epsilon)$ (d) $\frac{1}{2p} \int_{-\infty}^{+\infty} \exp(i|x - x^4|u) du$

12. The error function is defined by $\text{erf}(x) = \left(2/\sqrt{\pi}\right) \int_0^x \exp(-t^2) dt$

Choose the correct alternatives:

- (a) Error function is an even function (b) Error function is an odd function
 (c) $\text{erf}(0) = 1$ (d) $\text{erf}(\infty) = 1$

13. A wave $f(r, z, t) = J_0(\alpha r) \cos(\omega t - kz)$, where r and z refer to cylindrical coordinate system, satisfies the wave equation

$$\nabla^2 f - (1/v^2) \left(\partial^2 f / \partial t^2 \right) = 0$$

when

- (a) $v^2 = \omega^2/k^2$ (b) $v^2 = \omega^2 / (k^2 + \alpha^2)$ (c) $v^2 = \omega^2 / (k^2 - \alpha^2)$ (d) $v^2 = \omega^2/\alpha^2$

14. The volume element in the spherical polar coordinates is given by:

- (a) $r^2 \sin \theta d\theta d\phi dr$ (b) $r^2 \sin \theta \sin \phi d\theta d\phi dr$
 (c) $r \sin \theta d\theta d\phi dr$ (d) $r^2 d\theta d\phi dr$

15. From dimensional argument, the Fourier transform of Coulomb potential in three dimension is:

- (a) $\frac{C}{k^3}$ (b) $\frac{C}{k}$ (c) C (d) $\frac{C}{k^2}$

where C is a constant, and k is the magnitude of the wave vector.

16. Which of the following is a valid solution of the differential equation, $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$

- (a) $x^2 - y^2$ (b) $ax^2 - by^2$ (c) $x^2 + y^2$ (d) $x^4 + y^4$



17. The Dirac delta function $\delta(x)$ is defined as $\int_{-\infty}^{\infty} \delta(x) dx = 1$. Which of the following integration is **INCORRECTLY** evaluated?
- (a) $\int_0^{2\pi} \cos x \delta(x - \pi) dx = -1$ (b) $\int_{-2}^{+2} x^3 \delta(x + 1) dx = -1$
- (c) $\int_{-2}^2 (2x + 3) \delta(3x) dx = 1$ (d) $\int_0^2 (x^3 + 3x + 2) \delta(x - 1) dx = 0$
18. The nonzero independent components of an antisymmetric tensor A_{ij} of rank 2 in three dimensions are
- (a) A_{11}, A_{12}, A_{13} (b) A_{12}, A_{13}, A_{22}
- (c) A_{12}, A_{23}, A_{13} (d) A_{13}, A_{23}, A_{33}
19. Which of the following matrices is an element of the group $SU(2)$?
- (a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1+i}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \end{pmatrix}$ (c) $\begin{pmatrix} 2+i & i \\ 3 & 1+i \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$
20. The number of independent components of a symmetric and an antisymmetric tensor of rank 2 (in 3-dimensions) are, respectively,
- (a) 6, 6 (b) 9, 3 (c) 6, 3 (d) 3, 6
21. The trapezoidal, Simpson's 1/3 and Simpson's 3/8 rules are exact for polynomials of order
- (a) 1, 2, 3 respectively (b) 1, 3, 3 respectively
- (c) 1, 3, 4 respectively (d) 2, 3, 4 respectively
22. The sum of $1 - \frac{\pi^2}{2!4^2} + \frac{\pi^4}{4!4^4} - \frac{\pi^6}{6!4^6} + \dots$ is
- (a) Not convergent (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$
23. The integral $\int_{-1}^1 x^{-2} dx$ is:
- (a) 0 (b) 1 (c) 2 (d) ∞
24. $\int_{-1}^1 \sin mx \cos nx dx, (m \neq 0, n \neq 0)$ is:
- (a) δ_{mn} (b) 2 (c) $2mn\pi$ (d) 0



25. Given $f(x) = \frac{1}{1-x^2}$, $x < 1$, the series expansion for $f(x)$ is:
- (a) $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots$ (b) $1+x+\frac{x^2}{2}+\frac{x^3}{3}+\dots$
- (c) $1+x+x^2+x^3+\dots$ (d) $1+x^2+x^4+x^6+\dots$
26. Consider the function f such that $f(\alpha+\beta) = f(\alpha) + f(\beta)$ for any real α, β . If m and n are integers, then $f\left(\frac{m}{n}\right)$ is
- (a) $\frac{m}{n}f(1)$ (b) $\frac{m}{n}$ (c) $\frac{f(m)}{f(n)}$ (d) $f\left(\frac{1}{m}\right) + f\left(\frac{1}{n}\right)$
27. Given functions $\cos 2x, \sin^2 x, \cos^2 x$ are
- (a) Linearly independent (b) Linearly dependent
- (c) Orthogonal to each other (d) Unrelated to each other
28. The value of the integral $\int_0^{\infty} x e^{-x^2} dx$ is equal to
- (a) $\sqrt{\frac{\pi}{2}}$ (b) $\sqrt{\pi}$ (c) 0 (d) $\frac{1}{2}$
29. Under a small rotation $\delta\theta$ about the x -axis, the component of a second rank tensor Γ_{yz} transforms as
- (a) $\Gamma'_{yz} = (1+\delta\theta)\Gamma_{yz}$ (b) $\Gamma'_{yz} = \Gamma_{yz} + \delta\theta(\Gamma_{yy} - \Gamma_{zz})$
- (c) $\Gamma'_{yz} = \Gamma_{yz} + \delta\theta(\Gamma_{yx} - \Gamma_{zx})$ (d) $\Gamma'_{yz} = \Gamma_{yz} + \delta\theta\Gamma_{xx}$
30. In spherical polar coordinates $\vec{r} = (r, \theta, \varphi)$ the delta function $\delta(\vec{r}_1 - \vec{r}_2)$ can be written as
- (a) $\delta(r_1 - r_2)\delta(\theta_1 - \theta_2)\delta(\varphi_1 - \varphi_2)$
- (b) $\frac{1}{r_1^2}\delta(r_1 - r_2)\delta(\cos\theta_1 - \cos\theta_2)\delta(\varphi_1 - \varphi_2)$
- (c) $\frac{1}{|\vec{r}_1 - \vec{r}_2|^2}\delta(r_1 - r_2)\delta(\cos\theta_1 - \cos\theta_2)\delta(\varphi_1 - \varphi_2)$
- (d) $\frac{1}{r_1^2 \cos\theta_1}\delta(r_1 - r_2)\delta(\theta_1 - \theta_2)\delta(\varphi_1 - \varphi_2)$



31. The character table of C_{3v} , the group of symmetries of an equilateral triangle is given below

	$\chi^{(0)}$	$\chi^{(1)}$	$\chi^{(2)}$
$1C_1$	1	1	b
$3C_2$	1	a	c
$2C_3$	1	1	d

In the above C_1, C_2, C_3 denotes the three classes of C_{3v} , containing 1, 3 and 2 elements respectively, and $\chi^{(0)}, \chi^{(1)}$ and $\chi^{(2)}$ are the characters of the three irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}$ and $\Gamma^{(2)}$ of C_{3v} .

A. The entries a, b, c and d in this table are, respectively

- (a) 2, 1, -1, 0 (b) -1, 2, 0, -1 (c) -1, 1, 0, -1 (d) -1, 1, 1, -1

B. The reducible representation Γ of C_{3v} with character $\chi = (4, 0, 1)$ decomposes into its irreducible representations $\Gamma^{(0)}, \Gamma^{(1)}, \Gamma^{(2)}$ as

- (a) $2\Gamma^{(0)} + 2\Gamma^{(1)}$ (b) $\Gamma^{(0)} + 3\Gamma^{(1)}$ (c) $\Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)}$ (d) $2\Gamma^{(2)}$

32. The function $f(x)$ represents the nearest integer less than x , e.g. $f(3.14) = 3$.

The derivative of this function (for arbitrary x) will be given in terms of the integers n as $f'(x) =$

- (a) 0 (b) $\sum_n \delta(x-n)$ (c) $\sum_n |x-n|$ (d) $\sum_n f(x-n)$

33. Consider the integral, $\int_{-p}^{+p} \frac{dx}{\sqrt{x^2 - p^2}}$

where p is a constant. This integral has a real, nonsingular value if

- (a) $p < -1$ (b) $p > 1$ (c) $p = 1$ (d) $p \rightarrow 0$

34. The value of the real integral $I = \int_{-3}^{+3} \frac{dx}{x^2 + 3x + 2}$ is:

- (a) $\frac{3\pi}{2}$ (b) $\ln\left(\frac{2}{5}\right)$ (c) ∞ (d) 0

35. Let $\phi(x)$ be an arbitrary smooth function of x in $x_1 \leq x \leq x_2$. The condition that the integral $J = \int_{x_1}^{x_2} \left(\frac{d\phi}{dx}\right)^2 dx$ has an extremum is given by

- (a) $\frac{d^2\phi}{dx^2} > 0$ (b) $\frac{d\phi}{dx} = x$ (c) $\frac{d^2\phi}{dx^2} < 0$ (d) $\frac{d^2\phi}{dx^2} = 0$

36. Let $f(x) = e^{-x} \cos x$, what is the value of $f(x)$ at $x = 0.01$ up to four decimal places

- (a) 0.0001 (b) 0.0101 (c) 0.9900 (d) 0.9999

37. The value of the integral $\int_0^{\infty} x e^{-x^2} dx$ is equal to

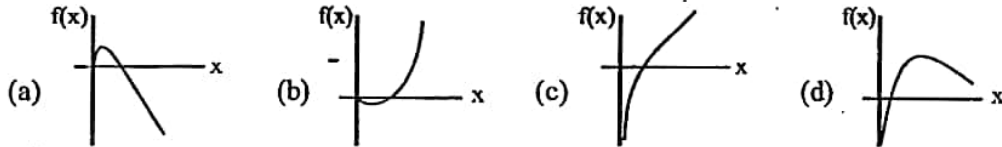
- (a) $\sqrt{\frac{x}{2}}$ (b) $\sqrt{x}$ (c) $\frac{1}{2}$ (d) 0



38. If $T_{ij} = \frac{1}{2}(x_i y_j - x_j y_i)$ and $S_{ij} = \frac{1}{2}(x_i y_j + x_j y_i)$ are components of tensor T and $S = (i, j = 1, 2, 3)$ then $\sum_{i,j} T_{ij} S_{ij}$ is:

(a) 0 (b) $\sum_i (x_i^2 - y_i^2)$ (c) $\sum_i x_i^2 y_i^2$ (d) $\sum_i (x_i^4 - y_i^4)$

39. Which of the following plots represents the function $f(x) = x \ln x$ in the range $0 \leq x < \infty$?

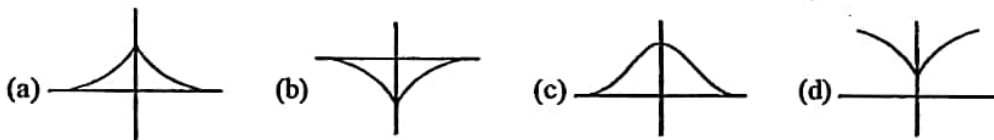


40. The values of a function $f(x)$ are tabulated for three values of x below:

x	$f(x)$
1.0	1.76
2.0	0.41
3.0	-0.16

What is the approximate value of x at which $f(x)$ vanishes?

- (a) 2.3 (b) 2.9 (c) 2.1 (d) 1.9
41. The number of independent components of symmetric second rank tensor S_{ij} , $i, j = 1, 2, \dots, d$ is
- (a) 2 (b) d^2 (c) $d^2/2$ (d) $d(d+1)/2$
42. The sum of the infinite series $1 + \frac{3}{2} + \frac{3.4}{2^2 \cdot 2!} + \frac{3.4.5}{2^3 \cdot 3!} + \frac{3.4.5.6}{2^4 \cdot 4!} + \dots$ is
- (a) 8 (b) e^2 (c) 2π (d) ∞
43. Which of the following plots represents the function $f(x) = e^{-|x|}$ for $-\infty < x < \infty$?



44. The maximum value of the function $f(x) = xe^{-x}$ is

(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) 1 (d) e^2

45. Given the integral $\int_0^{\infty} e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$, the value of the integral $\int_0^{\infty} x^2 e^{-2x^2} dx$ is

(a) $\frac{1}{4} \sqrt{\frac{\pi}{2}}$ (b) $2\sqrt{2\pi}$ (c) $2\sqrt{\pi}$ (d) 0



46. The numerical value of a function $f(x)$ at three points are tabulated below:

x	$f(x)$
0	0.0000
0.1	0.0998
0.2	0.1986

The approximate value $\frac{d^2f(x)}{dx^2}$ at $x=0.1$ is

- (a) -0.02 (b) -0.01 (c) -0.1 (d) +0.1
47. The number of independent components of the symmetric tensor A_{ij} with indices $i, j = 1, 2, 3$ is
(a) 1 (b) 3 (c) 6 (d) 9
48. As $x \rightarrow 1$, infinite series $x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \dots$
(a) diverges (b) converges to unity
(c) converges to $\pi/4$ (d) none of the above
49. If $[x]$ denotes the greatest integer not exceeding x , then $\int_0^e [x] e^{-x} dx$ is
(a) $\frac{1}{e-1}$ (b) 1 (c) $\frac{e-1}{e}$ (d) $\frac{e}{e^2-1}$
50. What is the value of the following series?
$$\left(1 + \frac{1}{2!} + \frac{1}{4!} + \dots\right)^2 - \left(1 + \frac{1}{3!} + \frac{1}{5!} + \dots\right)^2$$

(a) 0 (b) e (c) e^2 (d) 1
51. Which of the following limits exists?
(a) $\lim_{N \rightarrow \infty} \left(\sum_{m=1}^N \frac{1}{m} + \ln N \right)$ (b) $\lim_{N \rightarrow \infty} \left(\sum_{m=1}^N \frac{1}{m} - \ln N \right)$
(c) $\lim_{N \rightarrow \infty} \left(\sum_{m=1}^N \frac{1}{\sqrt{m}} - \ln N \right)$ (d) $\lim_{N \rightarrow \infty} \sum_{m=1}^N \frac{1}{m}$
52. A bag contain many balls, each with a number painted on it. There are exactly n balls which have the number n (namely one ball with 1, two balls with 2, and so on until N balls with N on them). An experiment consists of choosing a ball at random, noting the number on it and returning it to the bag. If the experiment is repeated a large number of times, the average value of the number will tend to
(a) $\frac{2N+1}{3}$ (b) $\frac{N}{2}$ (c) $\frac{N+1}{2}$ (d) $\frac{N(N+1)}{2}$
53. The value of the integral
$$\int_0^{\infty} dx x^9 \exp(-x^2)$$

is
(a) 20160 (b) 12 (c) 18 (d) 24



54. The integral

$$\int_{-\infty}^{\infty} dx \delta(x^2 - \pi^2) \cos x$$

evaluates to

- (a) -1 (b) 0 (c) $\frac{1}{\pi}$ (d) $-\frac{1}{\pi}$

55. What is the value of the following series? $\left(1 - \frac{1}{2!} + \frac{1}{4!} - \dots\right)^2 + \left(1 - \frac{1}{3!} + \frac{1}{5!} - \dots\right)^2$

- (a) 0 (b) e (c) e^2 (d) 1

56. If the distribution function of x is $f(x) = xe^{-x/\lambda}$ over the interval $0 < x < \infty$, the mean value of x is

- (a) λ (b) 2λ (c) $\lambda/2$ (d) 0

57. The expression

$$\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} + \frac{\partial^2}{\partial x_4^2} \right) \frac{1}{(x_1^2 + x_2^2 + x_3^2 + x_4^2)} \text{ is proportional to}$$

- (a) $\delta(x_1 + x_2 + x_3 + x_4)$ (b) $\delta(x_1)\delta(x_2)\delta(x_3)\delta(x_4)$
(c) $(x_1^2 + x_2^2 + x_3^2 + x_4^2)^{-3/2}$ (d) $(x_1^2 + x_2^2 + x_3^2 + x_4^2)^{-2}$

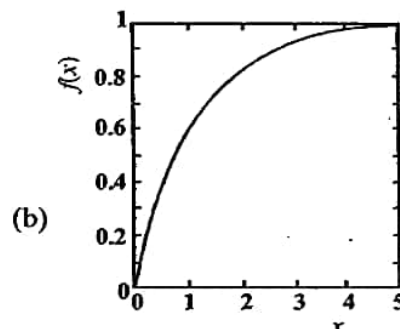
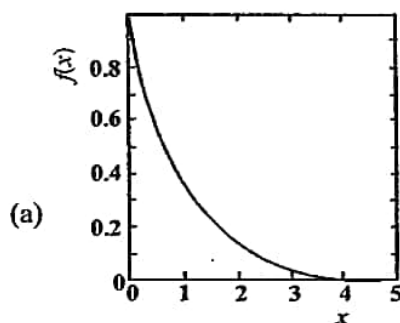
58. Let A and B be two vectors in three-dimensional Euclidean space. Under rotation, the tensor product

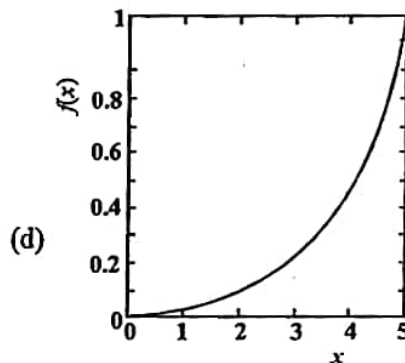
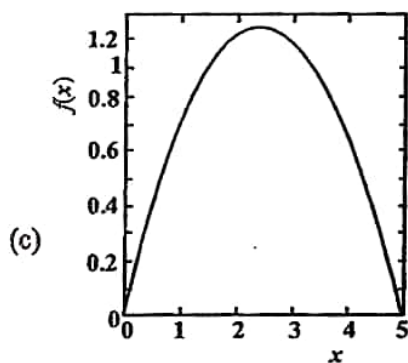
$$T_{ij} = A_i B_j$$

- (a) reduces to a direct sum of three 3-dimensional representations
(b) is an irreducible 9-dimensional representation
(c) reduces to a direct sum of a 1-dimensional, a 3-dimensional and a 5-dimensional irreducible representations
(d) reduces to a direct sum of a 1-dimensional and an 8-dimensional irreducible representation

59. The solution of the integral equation $f(x) = x - \int_0^x dt f(t)$

has the graphical form





60. In the most general case, which one of the following quantities is NOT a second order tensor?
 (a) Stress (b) Strain (c) Moment of inertia (d) Pressure
61. The following data is obtained in an experiment that measures the viscosity η as a function of molecular weight M for a set of polymers.

M (Da)	η (kPa-s)
990	0.28 ± 0.03
5032	30 ± 2
10191	250 ± 10
19825	2000 ± 200

The relation that best describes the dependence of η on M is

- (a) $\eta \sim M^{4/9}$ (b) $\eta \sim M^{3/2}$ (c) $\eta \sim M^2$ (d) $\eta \sim M^3$
62. The integral $\int_0^1 \sqrt{x} dx$ is to be evaluated up to 3 decimal places using Simpson's 3-point rule. If the interval $[0, 1]$ is divided into 4 equal parts, the correct result is
 (a) 0.683 (b) 0.667 (c) 0.657 (d) 0.638



Miscellaneous_Answer Key

1. (a)	2. (c)	3. (d)	4. (a)	5. (b)	6. (c)	7. (b)
8. (b)	9. (b)	10. (c)	11. (b)	12. (b,d)	13. (b)	14. (a)
15. (b)	16. (a)	17. (d)	18. (c)	19. (b)	20. (c)	21. (a)
22. (d)	23. (a)	24. (d)	25. (d)	26. (a)	27. (b)	28. (d)
29. (c)	30. (b)	31. (b), (c)	32. (b)	33. (d)	34. (d)	35. (d)
36. (c)	37. (c)	38. (a)	39. (b)	40. (b)	41. (d)	42. ()
43. (a)	44. (a)	45. (a)	46. (d)	47. (c)	48. (c)	49. (a)
50. (d)	51. (b)	52. (a)	53. (b)	54. (d)	55. (d)	56. (b)
57. (b)	59. (b)	60. (d)	61. (d)	62. (c)		





CAREER ENDEAVOUR

ACADEMY PRIVATE LIMITED

Institute for NET-JRF/GATE & IIT-JAM Exams

Subject: PHYSICAL SCIENCES (NET-JRF/GATE)

Section: MATHEMATICAL PHYSICS

Assignment 1: VECTOR ALGEBRA & VECTOR CALCULUS

- The angle between the vectors $2\hat{i} + 2\hat{j} - \hat{k}$ and $6\hat{i} - 3\hat{j} + 2\hat{k}$ is equal to
 (a) $\cos^{-1}(4/11)$ (b) $\sin^{-1}(4/\sqrt{11})$ (c) $\cos^{-1}(4/21)$ (d) $\sin^{-1}(4/\sqrt{21})$
- If the projection of the vector $\vec{A} = \hat{i} + m\hat{j} + \hat{k}$ on vector $\vec{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$ is equal to $19/9$, then the value of 'm' is equal to
 (a) 1 (b) -1 (c) 2 (d) -2
- For any vector $\vec{A}$, which of the following statements is true?
 (a) $\vec{A} = (\vec{A} \times \hat{i})\hat{i} + (\vec{A} \times \hat{j})\hat{j} + (\vec{A} \times \hat{k})\hat{k}$ (b) $\vec{A} = (\vec{A} \times \hat{i})\hat{i} + (\vec{A} \times \hat{j})\hat{j} + (\vec{A} \times \hat{k})\hat{k}$
 (c) $\vec{A} = (\vec{A} \cdot \hat{i})\hat{i} + (\vec{A} \cdot \hat{j})\hat{j} + (\vec{A} \cdot \hat{k})\hat{k}$ (d) $\vec{A} = (\vec{A} \cdot \hat{i})\hat{i} + (\vec{A} \cdot \hat{j})\hat{j} + (\vec{A} \cdot \hat{k})\hat{k}$
- Given: $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{c} = 2\hat{i} + \hat{j} - 4\hat{k}$ are the sides of
 (a) an isosceles triangle (b) a right-angled triangle
 (c) an equilateral triangle (d) none of these
- The value of the constant 'm' for which $\vec{A} = m\hat{i} + \hat{j} + \sqrt{5}\hat{k}$ subtends an angle 60° with $\vec{B} = 4\hat{i} - 5\hat{j} + \sqrt{5}\hat{k}$ is equal to
 (a) $\sqrt{46/3}$ (b) $\sqrt{29/4}$ (c) $\sqrt{41/3}$ (d) $\sqrt{17/3}$
- The range of x for which the angle between the vectors $\vec{A} = 2x^2\hat{i} + 4x\hat{j} + \hat{k}$ and $\vec{B} = 7\hat{i} - 2\hat{j} + x\hat{k}$ is obtuse, is equal to
 (a) $x < 0$ (b) $x > 1/2$ (c) $0 < x < 1/2$ (d) None of these
- If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, then the angle between $\vec{a}$ and $\vec{b}$ is
 (a) $\cos^{-1}(15/22)$ (b) $\cos^{-1}(225/2)$ (c) $\pi/2$ (d) $\pi/3$
- $|\vec{a} \times \vec{b}|$ can be written as
 (a) $a^2b^2 - (\vec{a} \cdot \vec{b})^2$ (b) $\sqrt{a^2b^2 - (\vec{a} \cdot \vec{b})^2}$ (c) $a^2b^2 + (\vec{a} \cdot \vec{b})^2$ (d) $a^2b^2 - \vec{a} \cdot \vec{b}$
- If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$, then which of the following statements is true?
 (a) $\vec{a}$ is parallel to $(\vec{b} - \vec{c})$ (b) $\vec{a}$ is perpendicular to $(\vec{b} - \vec{c})$
 (c) either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{c}$ (d) None of these



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10. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$, then $\vec{a} + t\vec{b}$ is perpendicular to $\vec{c}$ if 't' is equal to
 (a) 8 (b) 4 (c) 6 (d) 2
11. If $\vec{a}$ and $\vec{b}$ are unit vectors, which of the following statements is true?
 (a) $\sin \frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$ (b) $\sin \frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$ (c) $\cos \frac{\theta}{2} = \frac{1}{2}|\vec{a} + \vec{b}|$ (d) $\cos \frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$
12. The area of the triangle PQR having vertices P(1, 3, 2), Q(2, -1, 1), R(-1, 2, 3) is equal to
 (a) $\frac{1}{2}\sqrt{107}$ (b) $\frac{1}{2}\sqrt{77}$ (c) $\frac{1}{2}\sqrt{117}$ (d) $\frac{1}{2}\sqrt{117}$
13. Given : $\vec{P} = 2\hat{i} - 3\hat{j}$, $\vec{Q} = \hat{i} + \hat{j} - \hat{k}$, $\vec{R} = 3\hat{i} - \hat{k}$, the vector $\vec{V}$, which is orthogonal to $\vec{P}$ and $\vec{Q}$ and having unit scalar product with $\vec{R}$, can be estimated to be
 (a) $(3\hat{i} + 2\hat{j} + 5\hat{k})/4$ (b) $(3\hat{i} + 2\hat{j} - 5\hat{k})/4$ (c) $(3\hat{i} - 2\hat{j} + 5\hat{k})/4$ (d) $(3\hat{i} - 2\hat{j} - 5\hat{k})/4$
14. Given: $\vec{A} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{B} = 3\hat{i} + 5\hat{j} + 6\hat{k}$, $\vec{C} = -\hat{i} + 4\hat{j} + m\hat{k}$. The value of 'm' such that the vectors will be coplanar is equal to
 (a) 13/2 (b) 7/2 (c) 5/2 (d) 11/2
15. The equation of the plane that contains the points P(2, -1, 1), Q(3, 2, -1), R(-1, 3, 2) is equal to
 (a) $11x + 5y + 13z = 30$ (b) $11x + 7y + 9z = 30$
 (c) $4x + 7y - 9z = 30$ (d) $4x + 11y - 3z = 30$
16. The unit vector which is perpendicular to vector $2\hat{i} + \hat{j} - 3\hat{k}$ and lie in the plane of vectors $7\hat{i} - \hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} - 3\hat{k}$ is equal to
 (a) $(3\hat{i} - 3\hat{j} - \hat{k})/\sqrt{19}$ (b) $(3\hat{i} - 3\hat{j} + \hat{k})/\sqrt{19}$ (c) $(3\hat{i} - 3\hat{j} + \hat{k})$ (d) $(3\hat{i} - 3\hat{j} - \hat{k})$
17. If $\vec{b} = \hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k})$, then $\vec{b}$ can be simplified to
 (a) 0 (b) $\vec{a}$ (c) $2\vec{a}$ (d) None of these
18. Which of the following relations is true?
 (a) $(\vec{A} \times \vec{B}) \cdot (\vec{B} \times \vec{C}) \times (\vec{C} \times \vec{A}) = 0$ (b) $(\vec{A} \times \vec{B}) \cdot (\vec{B} \times \vec{C}) \times (\vec{C} \times \vec{A}) = (\vec{A} \cdot \vec{B} \times \vec{C})^2$
 (c) $(\vec{A} \times \vec{B}) \cdot (\vec{B} \times \vec{C}) \times (\vec{C} \times \vec{A}) = (\vec{A} \times (\vec{B} \times \vec{C}))^2$ (d) $(\vec{A} \times \vec{B}) \cdot (\vec{B} \times \vec{C}) \times (\vec{C} \times \vec{A}) = (\vec{A} \times (\vec{B} \times \vec{C}))$
19. Three unit vectors $\vec{a}, \vec{b}, \vec{c}$ ($\vec{b}$ and $\vec{c}$ are not parallel) are such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}\vec{c}$. The angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$, respectively are
 (a) $30^\circ, 90^\circ$ (b) $150^\circ, 90^\circ$ (c) $60^\circ, 90^\circ$ (d) $90^\circ, 30^\circ$



20. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. The components of its acceleration at $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$, is equal to
 (a) $2/\sqrt{14}$ (b) $-2/\sqrt{14}$ (c) $16/\sqrt{14}$ (d) $-16/\sqrt{14}$
21. The unit tangent vector on the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ at the point where $t = 2$ is equal to
 (a) $\frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$ (b) $\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$ (c) $\frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$ (d) $\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$
22. The unit vector perpendicular to the surface $x^2 + y^2 - z^2 = 11$ at the point $(4, 2, 3)$ is equal to
 (a) $4\hat{i} + 2\hat{j} + 3\hat{k} / \sqrt{29}$ (b) $4\hat{i} + 2\hat{j} - 3\hat{k} / \sqrt{29}$
 (c) $4\hat{i} + 2\hat{j} - 3\hat{k}$ (d) $4\hat{i} + 2\hat{j} + 3\hat{k}$
23. The equation of a surface of revolution is $z = \pm \sqrt{\frac{3}{2}x^2 + \frac{3}{2}y^2}$. The unit normal to the surface at the point $A\left(\sqrt{\frac{2}{3}}, 0, 1\right)$ is
 (a) $\sqrt{\frac{3}{5}}\hat{i} + \frac{2}{\sqrt{10}}\hat{k}$ (b) $\sqrt{\frac{3}{10}}\hat{i} + \frac{2}{\sqrt{10}}\hat{j}$ (c) $\sqrt{\frac{3}{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{k}$ (d) $\sqrt{\frac{3}{5}}\hat{i} - \frac{2}{\sqrt{10}}\hat{k}$
24. The angle between the two surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$ is
 (a) $\sin^{-1}\left(\frac{8}{3\sqrt{21}}\right)$ (b) $\cos^{-1}\left(\frac{8}{\sqrt{21}}\right)$ (c) $\sin^{-1}\left(\frac{8}{\sqrt{21}}\right)$ (d) $\cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$
25. The directional derivative of $1/r$ in the direction of $\vec{r}$ is equal to
 (a) $-1/r$ (b) $-1/r^2$ (c) $-1/r^3$ (d) $-1/r^4$
26. The maximum value of direction derivative of $\phi = x^2 - 2y^2 + 4z^2$ at the point $(1, 1, -1)$ is equal to
 (a) $6\sqrt{7/3}$ (b) $\sqrt{7/3}$ (c) 84 (d) $3\sqrt{3/7}$
27. The surface $ax^2 - byz = (a+2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$. Which of the following relations is true for 'a' and 'b'?
 (a) $a - b = 4$ (b) $2a - b = 4$ (c) $a = 3b$ (d) $3a + b = 9$
28. The directions along which there is no change in the value of the function $f(x, y) = e^{(x^2 + y^2)}$ at the point $(3, -2)$ are equal to
 (a) $0.6\hat{i} - 0.8\hat{j}$, $-0.6\hat{i} + 0.8\hat{j}$ (b) $-0.6\hat{i} - 0.8\hat{j}$, $0.6\hat{i} + 0.8\hat{j}$
 (c) $-0.6\hat{i} - 0.8\hat{j}$, $0.6\hat{i} - 0.8\hat{j}$ (d) $0.6\hat{i} + 0.8\hat{j}$, $-0.6\hat{i} + 0.8\hat{j}$



29. If $\vec{F}$ is a constant vector and $\vec{r}$ is the position vector then $\nabla(\vec{F} \cdot \vec{r})$ would be
 (a) $(\nabla \cdot \vec{r})\vec{F}$ (b) $|\vec{r}|\vec{F}$ (c) $(\nabla \cdot \vec{F})\vec{r}$ (d) $\vec{F}$
30. The value of 'm' for which $\vec{V} = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + mz)\hat{k}$ is a solenoidal vector is equal to
 (a) -1 (b) 1 (c) -2 (d) 2
31. If $\vec{a}$ and $\vec{b}$ are irrotational in nature, then $\vec{a} \times \vec{b}$ will be
 (a) irrotational (b) solenoidal (c) rotational (d) none of these
32. If $f = \tan^{-1}(y/x)$, then $\text{div}(\text{grad } f)$ is equal to
 (a) 1 (b) -1 (c) 0 (d) 2
33. The directional derivative of $\Omega(x, y) = \frac{x}{x^2 + y^2}$ along a line making an angle 30° with the positive x-axis at (0,2)
 (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{\sqrt{3}}{8}$ (d) $\frac{1}{2\sqrt{2}}$
34. For a constant vector $\vec{a}$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, consider the following statements:
 I. $\text{curl}(\vec{a} \times \vec{r}) = 2[\text{grad}(\vec{a} \cdot \vec{r})]$, II. $\text{div}[(\vec{a} \cdot \vec{r})\vec{r}] = 2(\vec{a} \cdot \vec{r})$, then
 (a) both statements are true (b) Only statement I is true
 (c) Only statement II is true (d) Both statements are false
35. The value of the line integral $\int \text{grad}(x + y + z) \cdot d\vec{r}$ from (0,1,-1) to (1,2,0) is
 (a) -1 (b) 3 (c) 0 (d) None of these
36. The work done by a force in moving a particle of mass 'm' from any point (x,y) to a neighbouring point (x+dx, y+dy) is given by $dW = 2xy dx + x^2 dy$. The work done for a complete cycle around a unit circle is
 (a) 0 (b) 1 (c) 3 (d) 2π
37. If $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$, then the value of the line integral $\int_C \vec{F} \cdot d\vec{r}$ along the curve C in the x-y plane given by $y = x^3$, from the point (1,1) to (2,8) is equal to
 (a) 35 (b) -35 (c) 47 (d) -47
38. If a force field $\vec{A} = (y + 2x)\hat{i} + (3x + 2y)\hat{j}$ is applied to a particle, then the work done by the force field in traversing the particle around a circle C in x-y plane, with center at the origin and radius 2 units is equal to (C is traverse in the counter-clockwise direction)
 (a) 2π (b) 4π (c) 8π (d) 16π
39. When a force $\vec{F} = (2x + y^2)\hat{i} + (3y - 4x)\hat{j}$ is applied on a particle, then it starts to move along a right angled triangle PQR having vertices P(0,0), Q(2,0), R(2,1) respectively. The amount of work done by the force is equal to
 (a) $14/3$ (b) $-14/3$ (c) $22/3$ (d) $-22/3$



40. The flux of the vector field $\vec{v} = r^2 \vec{r}$ over the region bounded by the surface S of a sphere of radius 'a', centered at the origin is equal to
- (a) πa^5 (b) $2\pi a^5$ (c) $4\pi a^5$ (d) $8\pi a^5$
41. If $\vec{A} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, then the value of the integral $\int_S \vec{A} \cdot \hat{n} dS$ (where S is the surface of a unit cube with two opposite corners at $(0,0,0)$ and $(1,1,1)$ respectively) is equal to
- (a) $3/2$ (b) $5/2$ (c) 6 (d) $9/2$
42. The value of the integral $\int_{(0,0)}^{(1,1)} [(3x^2 + 4xy + y^2)dx + (2x^2 + 2xy)dy]$ along the path $x^4 + y^4 = 2xy$ is
- (a) 0 (b) 1 (c) 2 (d) 4 *Green's theorem*
43. The volume of the portion of the cylinder $x^2 + y^2 = 4$ in the first octant between the planes $z = 0$ and $3x - z = 0$ is
- (a) 2 (b) 4 (c) 8 (d) 16
44. Let 'S' be the Boundary of the region consisting of the parabolic cylinder $z = 1 - x^2$ and the planes $y = 0, y = 2, z = 0$. The flux of the vector field $\vec{F} = xy\hat{i} + (y^2 + e^{xz})\hat{j} + \sin(xy)\hat{k}$ through the surface 'S' is
- (a) 2 (b) 4 (c) 8 (d) 16

CSIR Previous Year Questions

45. Let $\vec{a}$ and $\vec{b}$ be two distinct three-dimensional vectors. Then the component of $\vec{b}$ that is perpendicular to $\vec{a}$ is given by [CSIR June - 2011]
- (a) $\frac{\vec{a} \times (\vec{b} \times \vec{a})}{a^2}$ (b) $\frac{\vec{b} \times (\vec{a} \times \vec{b})}{b^2}$ (c) $\frac{(\vec{a} \cdot \vec{b})\vec{b}}{b^2}$ (d) $\frac{(\vec{b} \cdot \vec{a})\vec{a}}{a^2}$
46. The equation of the plane that is tangent to the surface $xyz = 8$ at the point $(1, 2, 4)$ is [CSIR Dec - 2011]
- (a) $x + 2y + 4z = 12$ (b) $4x + 2y + z = 12$
 (c) $x + 4y + 2z = 12$ (d) $x + y + z = 7$
47. A vector perpendicular to any vector that lies on the plane defined by $x + y + z = 5$, is [CSIR June - 2012]
- (a) $\vec{i} + \vec{j}$ (b) $\vec{j} + \vec{k}$ (c) $\vec{i} + \vec{j} + \vec{k}$ (d) $2\vec{i} + 3\vec{j} + 5\vec{k}$
48. The unit normal vector at the point $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$ on the surface of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, is [CSIR Dec - 2012]
- (a) $\frac{bci + caj + abk}{\sqrt{a^2 + b^2 + c^2}}$ (b) $\frac{ai + bj + ck}{\sqrt{a^2 + b^2 + c^2}}$ (c) $\frac{bi + cj + ak}{\sqrt{a^2 + b^2 + c^2}}$ (d) $\frac{i + j + k}{\sqrt{3}}$



49. A unit vector $\hat{n}$ on the xy -plane is at an angle of 120° with respect to $\hat{j}$. The angle between the vectors $\vec{u} = a\hat{i} + b\hat{n}$ and $\vec{v} = a\hat{n} + b\hat{i}$ will be 60° if [CSIR June - 2013]
- (a) $b = \sqrt{3}a/2$ (b) $b = 2a/\sqrt{3}$ (c) $b = a/2$ (d) $b = a$
50. If $\vec{A} = yz\hat{i} + xz\hat{j} + xy\hat{k}$, then the integral $\oint_C \vec{A} \cdot d\vec{l}$ (where C is along the perimeter of a rectangular area bounded by $x = 0$, $x = a$ and $y = 0$, $y = b$) is [CSIR Dec - 2013]
- (a) $\frac{1}{2}(a^3 + b^3)$ (b) $\pi(ab^2 + a^2b)$ (c) $\pi(a^3 + b^3)$ (d) 0
51. If $\vec{A} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ and C is the circle of unit radius in the plane defined by $z = 1$, with the centre on the z -axis, then the value of the integral $\oint_C \vec{A} \cdot d\vec{l}$ is [CSIR June - 2014]
- (a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

ANSWER KEY

- 1.(c) 2.(d) 3.(c) 4.(b) 5.(a) 6.(c) 7.(d) 8.(b) 9.(c) 10.(a)
- 11.(a)(c) 12.(a) 13.(a) 14.(a) 15.(a) 16.(b) 17.(c) 18.(b) 19.(b) 20.(b)
- 21.(c) 22.(b) 23.(d) 24.(d) 25.(b) 26.(a) 27.(b) 28.(a) 29.(d) 30.(c)
- 31.(b) 32.(c) 33.(c) 34.(b) 35.(b) 36.(a) 37.(a) 38.(c) 39.(b) 40.(c)
- 41.(a) 42.(a) 43.(c) 44.(c) 45.(a) 46.(b) 47.(c) 48.(a) 49.(c) 50.(d)
- 51.(d)





CAREER ENDEAVOUR

ACADEMY PRIVATE LIMITED

Institute for NET-JRF/GATE & IIT-JAM Exams

Subject: PHYSICAL SCIENCES (NET-JRF/GATE)

Section: MATHEMATICAL PHYSICS

Assignment 2: VECTOR ALGEBRA & VECTOR CALCULUS

- Find the unit vector normal to the plane of two vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$; and $\sin$ of the angle between the vectors.
[Ans. $(-3\hat{i} + 5\hat{j} + 11\hat{k})/\sqrt{155}; \sqrt{155/156}$]
- Show that $\vec{a} = [1/3, -2/3, -2/3]$ and $\vec{b} = [2/3, -1/3, 2/3]$ are orthogonal and find a vector $\vec{C}$ orthogonal to both $\vec{a}$ and $\vec{b}$.
[Ans. $\vec{C} = [-2/3, -2/3, 1/3]$]
- If the points P, Q, R are not lying on the same straight line having position vectors $\vec{x}, \vec{y}, \vec{z}$ relative to a given origin, show that $\vec{x} \times \vec{y} + \vec{y} \times \vec{z} + \vec{z} \times \vec{x}$ is perpendicular to the plane containing P, Q, R.
- Find a set of vectors reciprocal to the following set $(2\hat{i} + 3\hat{j} - \hat{k}), (\hat{i} - \hat{j} - 2\hat{k}), (-\hat{i} + 2\hat{j} + 2\hat{k})$,
[Ans. $(2\hat{i} + \hat{k})/3, (-8\hat{i} + 3\hat{j} - 7\hat{k})/3, (-7\hat{i} + 3\hat{j} - 5\hat{k})/3$]
- If $\vec{r} = \vec{a} \cos \omega t + \vec{b} \sin \omega t$, show that
(i) $\vec{r} \times \frac{d\vec{r}}{dt} = \omega (\vec{a} \times \vec{b})$ (ii) $\frac{d^2\vec{r}}{dt^2} = -\omega^2 \vec{r}$
[$\vec{a}, \vec{b}$ are constant vectors and ' ω ' is constant]
- If $\vec{A}$ has constant magnitude show that $\vec{A}$ and $\frac{d\vec{A}}{dt}$ are perpendicular to each other.
- A particle moves so that its position vector is given by, $\vec{r} = \cos \omega t \hat{i} + \sin \omega t \hat{j}$ where ' ω ' is constant. Show that,
(i) The velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$.
(ii) The acceleration is directed towards the origin and has magnitude proportional to the distance from the origin
(iii) $\vec{r} \times \vec{v} = \text{constant vector}$.
- If $\frac{d\vec{m}}{dt} = \vec{u} \times \vec{m}$ and $\frac{d\vec{n}}{dt} = \vec{u} \times \vec{n}$, then show that $\frac{d}{dt}(\vec{m} \times \vec{n}) = \vec{u} \times (\vec{m} \times \vec{n})$
- Find $\nabla \phi$ if (i) $\phi = \ln|\vec{r}|$, (ii) $\phi = \frac{1}{r}$ and (iii) $\phi = r^n$
[Ans. (i) $\frac{\vec{r}}{r^2}$, (ii) $-\frac{\vec{r}}{r^3}$]



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10. If $f = x + y + z$, $g = x^2 + y^2 + z^2$, $h = xy + yz + zx$, then show that $[\nabla f \nabla g \nabla h] = 0$.
11. Find the direction derivative of the scalar fn. $\phi = 4e^{(2x-y+z)}$ at the point $(1, +1, -1)$ in a direction towards the point $(-3, 5, 6)$. What will be its maximum value? [Ans. $-20/9, 4\sqrt{6}$]
12. If $\nabla U = 2r^4 \bar{r}$, find U , [Ans. $U = \frac{r^6}{3} + \text{constant}$]
13. Prove that, (i) $\nabla \cdot \left(\frac{\bar{r}}{r^3} \right) = 0$ ($r \neq 0$) (ii) $\nabla \cdot (\bar{r}^n \bar{r}) = (3+n)r^n$ (iii) $\nabla \left[\frac{\bar{a} \cdot \bar{r}}{r^n} \right] = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \cdot \bar{r})\bar{r}}{r^{n+2}}$
14. Prove that, (i) $\nabla \times (\bar{a} \times \bar{r}) = 2\bar{a}$ (ii) $\nabla \cdot (\hat{e} \bar{r}) = 1$
 (iii) $\nabla \times \left(\frac{\hat{k}}{r} \right) = \frac{-\hat{i}y + \hat{j}x}{r^3}$ (iv) $\nabla \cdot \{ \bar{a} \times (\bar{r} \times \bar{a}) \} = 2a^2$
 Given: $\hat{e}$ is an unit vector and $\bar{a}$ is a constant vector.
15. Find the values of a, b, c so that the function $\bar{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational [Ans. $a = 4, b = 2, c = -1$]
16. Show that, $\bar{v} = (x + 2y + 4z)\hat{i} + (2x - 3y - z)\hat{j} + (4x - y + 2z)\hat{k}$ is irrotational. Find the corresponding scalar potential ' ϕ '. [Ans. $\phi = \frac{x^2}{2} - \frac{3y^2}{2} + z^2 + 2xy + 4xz - yz$]
17. Prove that, if a rigid body is rotating with a constant angular velocity, then the curl of linear velocity at any point gives twice its angular velocity.
18. Show that $r^n \bar{r}$ is irrotational vector for any value of n , but it is solenoidal for $n = -3$ only.
19. Find an equation for the tangent plane to the surface $2xz^2 - 3xy - 4x = 7$ at the point $(1, -1, 2)$. [Ans. $7x - 3y + 8z = 26$]
20. Given: $\bar{r}(t) = 2\hat{i} - \hat{j} + 2\hat{k}$ when $t = 2$ and $\bar{r}(t) = 4\hat{i} - 2\hat{j} + 3\hat{k}$ when $t = 3$. Show that, $\int_2^3 \bar{r} \cdot \frac{d\bar{r}}{dt} dt = 10$
21. Evaluate $\int \bar{F} \cdot d\bar{r}$ where $\bar{F} = C[-3a \sin \theta \cos \theta \hat{i} + a(2 \sin \theta - 3 \sin^2 \theta)\hat{j} + b \sin 2\theta \hat{k}]$ and the curve is given by, $\bar{r} = a \cos \theta \hat{i} + a \sin \theta \hat{j} + b\theta \hat{k}$ (θ is varying from $\frac{\pi}{4}$ to $\frac{\pi}{2}$) [Ans. $\frac{1}{2}c(a^2 + b^2)$]
22. The acceleration of a particle at any time $t \geq 0$ is given by, $\bar{a} = 12 \cos 2t \hat{i} - 8 \sin 2t \hat{j} + 16t \hat{k}$
 If velocity $\bar{v}$ and displacement $\bar{r}$ are zero, at $t = 0$; find $\bar{v}$ and $\bar{r}$ at any time.
 [Ans. (i) $6 \sin 2t \hat{i} + (4 \cos 2t - 4)\hat{j} + 8t^2 \hat{k}$; (ii) $(3 - 3 \cos 2t)\hat{i} + (2 \sin 2t - 4t)\hat{j} + \frac{8}{3}t^3 \hat{k}$]
23. If $\bar{A} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, evaluate $\int \bar{A} \cdot d\bar{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the following paths:
 (i) $x = t, y = t^2, z = t^3$.



(ii) The straight lines from $(0, 0, 0) \rightarrow (1, 0, 0) \rightarrow (1, 1, 0) \rightarrow (1, 1, 1)$

(iii) The straight line joining $(0, 0, 0)$ and $(1, 1, 1)$

[Ans. (i) 5, (ii) $23/3$, (iii) $13/3$]

24. Find $I = \int (x dy - y dx)$ over the

(i) Straight line $y = x$ from $(0, 0)$ to $(1, 1)$

(ii) Parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$

(iii) Circle $x = a \cos t, y = 1 + \sin t; \frac{-\pi}{2} \leq t \leq 0$

[Ans. (i) 0, (ii) $1/3$, (iii) $a \left(\frac{\pi}{2} - 1 \right)$]

25. Show that, $\oint_C \left[-\frac{y}{x^2 + y^2} \hat{i} + \frac{x}{x^2 + y^2} \hat{j} \right] \cdot d\vec{r} = 2\pi$, where 'C' is the circle $x^2 + y^2 = 1$ in the x-y plane, described in counter clockwise direction.

26. Find $\int \vec{F} \cdot d\vec{r}$ along the curve defined by $x^2 = 4y$ and $3x^2 = 8z$ from $x = 0$ to $x = 2$, where $\vec{F} = 3x^2 \hat{i} + (2xz - y) \hat{j} + z \hat{k}$

[Ans. $441/40$]

27. Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy \hat{i} - 5z \hat{j} + 10x \hat{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$

[Ans. 303]

28. (i) Show that, $\vec{F} = (2xy + z^3) \hat{i} + x^2 \hat{j} + 3xz^2 \hat{k}$ is a conservative force fields.

(ii) Find the scalar potential.

(iii) Find the work done in moving an object in this field from $(1, -2, 1)$ to $(3, 1, 4)$

[Ans. (ii) $x^2 y + x z^3 + C$, (iii) 202 units]

29. Find the circulation of the field $\vec{F} = -x^2 y \hat{i} + x y^2 \hat{j} + (y^3 - x^3) \hat{k}$, around the curve the curve C, where C is the intersection of the sphere $x^2 + y^2 + z^2 = 25$ and the plane $z = 3$.

[Ans. 128π]

30. If 'f' is a scalar function which is a solution of laplace equation $\nabla^2 f = 0$ in a volume 'V' bounded by a piecewise smooth surface 'S', then apply Gauss divergence theorem to $\vec{u} = \nabla f$ to prove that, $\oint_S \vec{n} \cdot \nabla f \, ds = 0$

31. Using Gauss divergence theorem, evaluate $\iiint_S (x^3 dy dz + y^3 dz dx + z^3 dx dy)$, where 'S' is the surface of the

sphere $x^2 + y^2 + z^2 = a^2$

[Ans. $\frac{12}{5} \pi a^5$]

32. Consider the following vector field: $\vec{V} = xz^2 \hat{i} - yz^2 \hat{j} + z(x^2 - y^2) \hat{k}$

(a) Calculate the volume integral of the divergence of $\vec{V}$ over the region defined by $-a \leq x \leq a, -b \leq y \leq b$ and $0 \leq z \leq c$.

(b) Calculate the flux of $\vec{V}$ out of the region through the surface at $z = c$.

(c) Hence deduce the net flux through the rest of the boundary of the region.



[Ans. (a) $\frac{4}{3}abc(a^2 - b^2)$, (b) $\frac{4}{3}abc(a^2 - b^2)$, (c) 0]

33. Calculate the flux of the vector field $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ through the surface bounded by the region $x^2 + y^2 = 4, z = 0, z = 3$. [Ans. 84π]

34. Evaluate $\iint_S (\nabla \times \vec{F}) \cdot d\vec{s}$ for $\vec{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where 'S' is upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and 'C' is its boundary. [Ans. π]

35. Evaluate $\iint_S (\nabla \times \vec{F}) \cdot d\vec{s}$, where $\vec{F} = (3x - 2y)\hat{i} + x^2z\hat{j} + y^2(z + 1)\hat{k}$ for a plane rectangular area with vertices at (0, 0), (1, 0), (1, 2), (0, 2) in the xy plane. [Ans. 4]

36. (a) Consider a constant vector field $\vec{v} = v_0\hat{k}$. Find any one of the many possible vectors $\vec{u}$ for which $\nabla \times \vec{u} = \vec{v}$.
(b) Calculate the flux associated with the field $\vec{v}$ through the hemispherical surface defined by

$$x^2 + y^2 + z^2 = r^2, z > 0 \quad [\text{Ans. (a) } v_0 x\hat{j}, \text{ (b) } \pi r^2 v_0]$$

37. Evaluate $\int_C (x^2 y dx + x^2 dy)$, where 'C' is boundary described counter-clockwise direction of the triangle with vertices (0, 0), (1, 0), (1, 1) [Ans. 5/12]

38. Evaluate $\oint_C (xy + y^2) dx + x^2 dy$, where 'C' is the closed curve of the region bounded by $y = x^2$ and $y = x$ [Ans. -1/12]

39. Evaluate $\oint_C [(2x^2 - y^2) dx + (x^2 + y^2) dy]$, where 'C' is the boundary of the surface enclosed by x-axis and the semicircle $y = (1 - x^2)^{1/2}$. [Ans. 4/3]

40. Find the mass of a metallic body of varying density $\rho(x, y, z) = \rho_0 + xyz$, bounded by the planes $x = 0, y = 0, z = 0$ and the sphere $x^2 + y^2 + z^2 = a^2$ lying in the the first octant. [Ans. $\frac{1}{6}\rho_0\pi a^3 + \frac{a^6}{48}$]

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Subject: PHYSICAL SCIENCES (NET-JRF/GATE)

Section: MATHEMATICAL PHYSICS

Assignment-3: MATRIX ALGEBRA

1. Let a_{ij} denote the element of the i^{th} row and j^{th} column in a 3×3 determinant ($1 \leq i \leq 3, 1 \leq j \leq 3$)

and let $a_{ij} = -a_{ji}$ of every i and j . Then the determinant has all the principal diagonal elements as

Skew matrix

- (a) 1 (b) -1 (c) 0 (d) none

2. Which of the following statements is wrong?

- (a) Trace of a unit matrix is always a fixed natural number
(b) Trace of the sum of two matrices is equal to sum of their traces
(c) Trace of the scalar matrix is some multiple of its order
(d) $\text{Tr}(AB) = \text{Tr}(BA)$

HK dec

3. If A and B are matrices of order 3×2 and C is a matrix of order 2×3 , then which of the following matrices is not defined?

- (a) $A^T + B$ (b) A^T and B^T (c) $A^T + C$ (d) $B + C^T$

4. If $A = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$, then A^{10} equals

(a) $\begin{bmatrix} 1 & 10\alpha \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 10\alpha \\ 0 & 10 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & \alpha^{10} \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & \alpha^{10} \\ 0 & 10 \end{bmatrix}$

5. Consider the matrix $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then B^{1023} equals

(a) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

6. Which of the following is true for the matrix $\begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$?

- (a) It is an idempotent matrix (b) It is nilpotent matrix of order 3
(c) It is an involutory matrix (d) None of the above



7. $(x \ y) \begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 15$. This matrix equation represents

(a) a circle of radius $\sqrt{15}$ units

(b) an ellipse with semi-minor axis $\sqrt{5}$ units

~~(c) an ellipse with semi-major axis $\sqrt{5}$ units~~

(d) a hyperbola

8. Which of the following statements is true for the matrix $\begin{pmatrix} ab & b^2 \\ -a^2 & -ab \end{pmatrix}$?

~~(a) It is a idempotent matrix~~

(b) It is a nilpotent matrix of order 2

(c) It is an involutory matrix

(d) It is a periodic matrix of order 2

9. The product of the two matrices $\begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{pmatrix}$ and $\begin{pmatrix} \cos^2 \phi & \cos \phi \sin \phi \\ \cos \phi \sin \phi & \sin^2 \phi \end{pmatrix}$ is zero. Which of the following statements is true?

~~(a) $\theta - \phi$ is odd multiple of $(\pi/2)$~~

(b) $\theta - \phi$ is even multiple of $(\pi/2)$

(c) $\theta - \phi$ is odd multiple of π

(d) $\theta - \phi$ is even multiple of π

10. If each element of ' $k \times k$ ' square matrix A can be chosen as a Boolean Variable, then one can construct 'M' number of different combinations of matrix 'A'. Therefore, 'M' is equal to

(a) $2k$

~~(b) 2^{k^2}~~

(c) k^2

(d) 2^k

11. If A and B $n \times n$ square symmetric matrices, then AB will be symmetric in nature if

~~(a) A and B both are triangular~~

(b) A and B both are non-singular

(c) A and B both commute

(d) A and B both anti-commute

12. The symmetric part of $P = \begin{pmatrix} a \\ b \end{pmatrix} (a-2 \ b)$ is

(a) $\begin{pmatrix} a^2-2 & ab-1 \\ ab-1 & b^2-2 \end{pmatrix}$

(b) $\begin{pmatrix} a(a-2) & b \\ b & b^2 \end{pmatrix}$

(c) $\begin{pmatrix} a(a-1) & b(a-1) \\ b(a-1) & b^2 \end{pmatrix}$

~~(d) $\begin{pmatrix} a(a-2) & b(a-1) \\ b(a-1) & b^2 \end{pmatrix}$~~

13. To convert a Hermitian matrix into skew-Hermitian one, the Hermitian matrix must be multiplied by

(a) -1

~~(b) i~~

(c) -i

(d) None of the above

14. Suppose the matrix B^+AB is Hermitian in nature. Which of the following statement is true?

~~(a) A is a hermitian matrix~~

(b) B is a hermitian matrix

(c) A is a skew hermitian matrix

(d) B is a skew hermitian matrix



15. Given two $(n \times n)$ matrices $\hat{P}$ and $\hat{Q}$ such that $\hat{P}$ is hermitian and $\hat{Q}$ is skew hermitian. Which one of the following combinations of $\hat{P}$ and $\hat{Q}$ is necessarily a hermitian matrix?
- (a) $\hat{P}\hat{Q}$ (b) $i\hat{P}\hat{Q}$ (c) $\hat{P} + i\hat{Q}$ (d) $\hat{P} + \hat{Q}$
16. Which of the following statements is true?
- (a) Product of two orthogonal matrices A and B will be orthogonal if A and B commutes
 (b) Product of two orthogonal matrices A and B will be always orthogonal
 (c) Product of two orthogonal matrices A and B will be never orthogonal
 (d) None of these
17. The matrix $\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is
- (a) Symmetric (b) Skew Hermitian (c) Orthogonal (d) Unitary
18. If 'I' is an identity matrix of order 'k x k' and $\text{adj}(2I) = 2^k I$, then 'k' is equal to
- (a) 1 (b) 2 (c) n-1 (d) n
19. Suppose A is $n \times n$ square invertible matrix. Which of the following statements is true for $A(\text{adj } A)$?
- (a) It is a null matrix of order 'n' (b) It is a unit matrix of order 'n'
 (c) It is a scalar matrix of order 'n' (d) None of these
20. If 'A' is 4×4 matrix with determinant 5, then the determinant of $\text{adj}(A)$ is equal to
- (a) 5 (b) 25 (c) 125 (d) 625
21. If 'A' is a 3×3 matrix with determinant 2, then the determinant of $\text{adj}[\text{adj}[\text{adj}(A^{-1})]]$ is equal to
- (a) $\frac{1}{512}$ (b) $\frac{1}{1024}$ (c) $\frac{1}{128}$ (d) $\frac{1}{256}$
22. For which value of λ , the following matrix $A = \begin{bmatrix} 0 & -4 & 1 \\ 2 & \lambda & -3 \\ 1 & 2 & -1 \end{bmatrix}$ will be invertible in nature?
- (a) $\lambda = 4$ (b) $\lambda \neq 8$ (c) $\lambda \neq 4$ (d) none
23. If 'U' is unit matrix and the matrix 'A' is defined as $A = I - 2UU^T$, then A^{-1} is equal to
- (a) $I - 2UU^T$ (b) $I + 2UU^T$ (c) $2UU^T - I$ (d) $4UU^T$
24. If a square matrix A has an eigenvalue λ , then the eigenvalue of the matrix $(kA)^T$ (where $k \neq 0$) is
- (a) λ/k (b) k/λ (c) λk (d) None of these



25. The eigenvalues of the matrix $\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ are

(a) 1, 3, 1

~~(b) 2, 3, 2~~

(c) 5, 1, 1

(d) 3, 5, 2

26. Two eigen values of the matrix $\begin{bmatrix} 1 & 0 & 0 & -\frac{a}{2} \\ 0 & 1 & 0 & -\frac{a}{2} \\ 0 & 0 & 1 & -\frac{a}{2} \\ 0 & 0 & 0 & a \end{bmatrix}$ are

(a) 1 and a

~~(b) 1 and -a~~

(c) $\frac{1}{2}$ and $-\frac{a}{2}$

(d) $\frac{1}{2}$ and $\frac{a}{2}$

27. A matrix is given by $M = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix}$. The eigenvalues of M are

~~(a) real and positive~~

(b) purely imaginary with modulus 1

(c) real and negative

(d) complex number with modulus 1

28. Consider a matrix $M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$. A 3-dimensional basis formed by eigenvectors of M is

(a) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ ~~(b) $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$~~ (c) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

29. The eigenvalues of $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ are

(a) (1, i, 1, i)

(b) (1, -1, 0, 0)

(c) (1, 1, 1, 1)

(d) (1, 1, i, -i)

30. The trace of the matrix of a 2 x 2 matrix is 1 and determinant 1. Which of the following has to be true?

(a) one of the eigenvalues is 0

(b) one of the eigenvalue is 1

(c) both of the eigenvalues are 1

(d) neither of the eigenvalues are 1

$\lambda_1 \lambda_2 = 1$
 $\lambda_1 + \lambda_2 = 1$



31. For which of the following matrices both eigenvalues are positive?

(a) $\begin{pmatrix} -7 & 2 \\ 2 & -7 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 4 & -3 \\ -3 & 5 \end{pmatrix}$

(d) $\begin{pmatrix} -1 & 4 \\ 4 & -1 \end{pmatrix}$

32. For an $N \times N$ matrix consisting of all ones,

(a) all eigenvalues are 1

(b) all eigenvalues are 0

(c) eigenvalues are 1, 2, 3, ..., N

(d) one eigenvalue is N and others are 0

33. If trace of matrix M is 11 and its determinant is 36, then the largest eigenvalue of M is equal to

(a) 4

(b) 6

(c) 9

(d) 12

34. A 2×2 matrix 'A' has eigenvalues $e^{in/4}$ and $e^{in/5}$ such that $A^n = I$. The smallest possible value of 'n' is

(a) 20

(b) 30

(c) 40

(d) 80

35. The degenerate eigenvalues of the matrix $M = \begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$ is:

(a) 2

(b) 3

(c) 4

(d) 5

36. Let A be a 3×3 hermitian matrix which satisfies the matrix equation $M^2 - 7M + 12I = 0$, where I refers to the identity matrix. What is the determinant of the matrix M given that the trace is 10?

(a) 27

(b) 36

(c) 48

(d) 64

37. Consider a 3×3 matrix of the following form:

$$\begin{pmatrix} a_1^2 & a_1 a_2 & a_1 a_3 \\ a_1 a_2 & a_2^2 & a_2 a_3 \\ a_1 a_3 & a_2 a_3 & a_3^2 \end{pmatrix}$$

The number of zero eigenvalues for this matrix is

(a) 0

(b) 1

(c) 2

(d) 3

38. The eigenvalues of the matrix $A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ are

(a) $\{1, 1, 1, 1\}$

(b) $\{1, 1, -1, -1\}$

(c) $\{0, 0, \sqrt{2}, -\sqrt{2}\}$

(d) $\{\sqrt{2}, -\sqrt{2}, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\}$



39. The eigen values of $(A^4 + 3A - 2I)$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$, are
 (a) 2, 20, 88 (b) 1, 2, 3 (c) 2, 20, 3 (d) 1, 20, 88
40. If a matrix A is such that $3A^3 + 2A^2 + 5A + I = 0$, then A^{-1} is equal to
 (a) $-(3A^2 + 2A + 5)$ (b) $3A^2 + 2A + 5$ (c) $3A^2 - 5A - 5$ (d) None of these
41. Let, 'M' be a $n \times n$ diagonalizable matrix which satisfies the equations $M^2 = M$, then $\text{Det}(M)$ is equal to
 (a) 0 (b) 1 (c) n (d) $n-1$
42. Let, 'P' be a 3×3 Hermitian matrix which satisfies the matrix equation $M^2 - 5M + 6I = 0$. Which of the following are the possible eigenvalues of M?
 (a) (0,1,2) (b) (0,2,3) (c) (1,3,4) (d) (2,3,4)
43. The matrix $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ can be related by a similarity transformation to a matrix
 (a) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 1 & 0 \\ 1 & -1 & -1 \\ 0 & 1 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$
44. Two different 2×2 matrices A and B found to be have same eigenvalues. It is then correct to state that $A = SBS^{-1}$ where S can be a
 (a) arbitrary 2×2 matrix (b) hermitian 2×2 matrix
 (c) traceless 2×2 matrix (d) unitary 2×2 matrix
45. The trace of a real 4×4 matrix $U = \exp(A)$, where $A = \begin{pmatrix} 0 & 0 & 0 & \frac{\pi}{4} \\ 0 & 0 & -\frac{\pi}{4} & 0 \\ 0 & \frac{\pi}{4} & 0 & 0 \\ -\frac{\pi}{4} & 0 & 0 & 0 \end{pmatrix}$ is equal to
 (a) $2\sqrt{2}$ (b) $\frac{\pi}{4}$ (c) 0 (d) 2



46. The rank of the matrix $\begin{pmatrix} \Omega & -1 & 0 \\ 0 & \Omega & -1 \\ -1 & 0 & \Omega \end{pmatrix}$ is 2, for Ω equal to

- (a) 0 (b) 1 (c) 2 (d) 4

47. The rank of matrix $\begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{bmatrix}$ (a, b, c , being all real), is 3 if

- (a) $a = b = c$
 (b) a, b, c are all different but $a + b + c = 0$
 (c) Two of the numbers a, b, c are equal but are different from the third.
 (d) a, b, c are all different and $a + b + c \neq 0$

48. The values of a and b for which the following systems of linear equation.

$$ax + y + 3z = a$$

$$2x + by - z = 3$$

$$5x + 7y + z = 7$$

has an infinite number of solutions, are

- (a) $a = 1, b = 1$ (b) $a = 1, b = 3$ (c) $a = 2, b = 3$ (d) $a = 2, b = 1$

49. The value of ' a ' for which the system of equations

$$a^3x + (a+1)^3y + (a+2)^3z = 0$$

$$ax + (a+1)y + (a+2)z = 0$$

$$x + y + z = 0$$

has a non-zero solution, is equal to

- (a) 1 (b) 0 (c) -1 (d) None of these

CSIR Previous Year Questions

Consider the matrix $M = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

[CSIR June - 2011]

50. The eigenvalues of M are

- (a) 0, 1, 2 (b) 0, 0, 3 (c) 1, 1, 1 (d) -1, 1, 3

51. The exponential of M simplifies (I is the 3×3 identity matrix)

(a) $e^M = I + \left(\frac{e^3 - 1}{3} \right) M$

(b) $e^M = I + M + \frac{M^2}{2!}$

(c) $e^M = I + 3^3 M$

(d) $e^M = (e - 1)M$



52. A 3×3 matrix M has $\text{Tr}[M] = 6, \text{Tr}[M^2] = 26, \text{Tr}[M^3] = 90$. Which of the following can be possible set of eigenvalues of M ? [CSIR Dec - 2011]

(a) $\{1, 1, 4\}$ (b) $\{-1, 0, 7\}$ (c) $\{-1, 3, 4\}$ (d) $\{2, 2, 2\}$

53. The eigenvalues of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ [CSIR June - 2012]

(a) $(1, 4, 9)$ (b) $(0, 7, 7)$ (c) $(0, 1, 13)$ (d) $(0, 0, 14)$

54. The eigenvalues of the anti-symmetric matrix $A = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$ where n_1, n_2, n_3 are components of an unit vector, are [CSIR June - 2012]

(a) $0, -i, i$ (b) $0, 1, -1$ (c) $0, 1+i, -1-i$ (d) $0, 0, 0$

55. A 2×2 matrix 'A' has eigenvalues $e^{i\pi/5}$ and $e^{i\pi/6}$. The smallest value of 'n' such that $A^n = I$ [CSIR Dec - 2012]

(a) 20 (b) 30 (c) 60 (d) 120

56. Given a 2×2 unitary matrix satisfying $U'U = UU' = I$ with $\det U = e^{i\varphi}$, one can construct a unitary matrix V ($V'V = VV' = I$) with $\det V = 1$ from it by [CSIR Dec - 2012]

(a) Multiplying U by $e^{-i\varphi/2}$ (b) Multiplying a single element of U by $e^{-i\varphi}$
(c) Multiplying any row or column by $e^{-i\varphi/2}$ (d) Multiplying U by $e^{-i\varphi}$

57. Consider a $n \times n$ ($n > 1$) matrix A , in which A_{ij} is the product of the indices i and j (namely $A_{ij} = ij$) The matrix A [CSIR Dec - 2013]

(a) has one degenerate eigenvalue with degeneracy $(n-1)$
(b) has two degenerate eigenvalues with degeneracies 2 and $(n-2)$
(c) has one degenerate eigenvalues with degeneracy n
(d) does not have any degenerate eigenvalues

58. Consider the matrix

$$M = \begin{pmatrix} 0 & 2i & 3i \\ -2i & 0 & 6i \\ -3i & -6i & 0 \end{pmatrix}$$

The eigenvalues of M are [CSIR June - 2014]

(a) $-5, -2, 7$ (b) $-7, 0, 7$ (c) $-4i, 2i, 2i$ (d) $2, 3, 6$



59. The matrices

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

satisfy the commutation relation

[CSIR June - 2014]

(a) $[A, B] = B + C, [B, C] = 0, [C, A] = B + C$

(b) $[A, B] = C, [B, C] = A, [C, A] = B$

(c) $[A, B] = B, [B, C] = 0, [C, A] = A$

(d) $[A, B] = C, [B, C] = 0, [C, A] = B$

ANSWER KEY

1.(c) 2.(a) 3.(a) 4.(a) 5.(b) 6.(c) 7.(c) 8.(b) 9.(a) 10.(b)

11.(c) 12.(d) 13.(b)(c) 14.(a) 15.(c) 16.(b) 17.(d) 18.(c) 19.(c) 20.(c)

21.(d) 22.(b) 23.(a) 24.(c) 25.(c) 26.(a) 27.(d) 28.(b) 29.(b) 30.(d)

31.(c) 32.(d) 33.(b) 34.(c) 35.(d) 36.(b) 37.(c) 38.(b) 39.(a) 40.(d)

41.(a) 42.(b)(d) 43.(d) 44.(d) 45.(a) 46.(b) 47.(d) 48.(b) 49.(c) 50.(b)

51.(a) 52.(c) 53.(d) 54.(a) 55.(c) 56.(a) 57.(a) 58.(b) 59.(d)

60. Determine the values of a, b, c such that $\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal.

61. Find the eigenvalues and normalized eigen vectors of the following matrices:

(i) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(ii) $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

(iii) $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

62. Find the characteristic equation, of the following matrix and verify Cayley-Hamilton theorem.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix}$$



63. Show that, $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ satisfies his own eigen equation and hence find A^{-1} .

64. Evaluate M^{50} for the matrix, $M = \begin{bmatrix} 4/3 & \sqrt{2}/3 \\ \sqrt{2}/3 & 5/3 \end{bmatrix}$

65. Diagonalise the following matrices:

(i) $\begin{bmatrix} 4/3 & \sqrt{2}/3 \\ \sqrt{2}/3 & 5/3 \end{bmatrix}$

(ii) $\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

66. Evaluate e^A for: (i) $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, (ii) $\begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$

67. If α is the characteristic root of a non-singular matrix, then show that, $\frac{|A|}{\alpha}$ is a characteristic root of (adj. A).

68. Represent the following transformations

$$x_1 = 3y_1 + 2y_2, x_2 = -y_1 + 4y_2, y_1 = z_1 + 2z_2, y_2 = 3z_1$$

by the use of matrices and find the composite transformation which expresses x_1, x_2 in terms of z_1, z_2 .

69. Consider the following transformation in three dimensions:

$$\begin{aligned} x' &= x \cos \theta + y \sin \theta \\ y' &= -x \sin \theta + y \cos \theta \\ z' &= z \end{aligned}$$

(i) Write down the transformation matrix [A].

(ii) Show that, $[A(\theta_1)] \times [A(\theta_2)] = [A(\theta_1 + \theta_2)]$

(iii) Is $A(\theta)$ is unitary?

(iv) What are the eigenvalues of $A(\theta)$?

70. Find the Characteristic equation of the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ and hence find

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I.$$

71. The matrix $A = \begin{pmatrix} p & r \\ r & q \end{pmatrix}$ is transformed to the diagonal form $D = M^{-1}AM$, where

$$M = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix}. \text{ Find the value of } \phi \text{ which gives the diagonal transformation.}$$





SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

SECTION: MATHEMATICAL PHYSICS

Assignment-4: COMPLEX ANALYSIS

1. If $x + iy = (1 - i\sqrt{3})^{100}$, then the co-ordinate of point $P(x, y) =$
 (a) $(2^{99}, 2^{99}\sqrt{3})$ (b) $(2^{99}, -2^{99}\sqrt{3})$ (c) $(-2^{99}, 2^{99}\sqrt{3})$ (d) none of these
2. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n$ is real
 (a) 3 (b) 2 (c) 4 (d) none of these
3. The sum of the following series:
 $i^2 + i^4 + i^6 + \dots$ up to $(2n + 1)$ terms
 will be
 (a) i (b) $-i$ (c) 1 (d) -1
4. If $1, \omega, \omega^2$ are the cube roots of unity, then $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5$ will be
 (a) 4 (b) 8 (c) 16 (d) 32
5. Multiplying a complex number z by $1 + i$ rotates the radius vector of z by an angle of
 (a) 90° clockwise (b) 45° anticlockwise
 (c) 45° clockwise (d) 90° anticlockwise
6. If $1, \omega, \omega^2$ be cube roots of unity, then the sum of the following series:
 $(1 - \omega + \omega^2)(1 - \omega^2 + \omega^4)(1 - \omega^4 + \omega^8) \dots$ to $2n$ factors
 will be
 (a) $2n$ (b) 2^{2n} (c) 0 (d) 1
7. If $z = (\lambda + 3) + i\sqrt{5 - \lambda^2}$ (λ is a real parameter and $i = \sqrt{-1}$), then the locus of z will be
 (a) circle (b) ellipse (c) parabola (d) hyperbola
8. If sum and products of two complex numbers are real, then which of the following statements is/are true?
 (a) both complex numbers are real (b) both complex numbers are purely imaginary
 (c) one of the complex numbers are real (d) both are complex conjugate of each other
9. The expression $\sinh^{-1} x$ can be simplified to
 (a) $\ln(x + \sqrt{x^2 + 1})$ (b) $\ln(x + \sqrt{x^2 - 1})$ (c) $\ln(x - \sqrt{x^2 - 1})$ (d) $\ln(x - \sqrt{x^2 + 1})$



10. The value of the following limit:

$$\lim_{z \rightarrow 0} \frac{\operatorname{Re}(z^2) + \operatorname{Im}(z^2)}{z^2}$$

- (a) 0 (b) 1 (c) -1 (d) does not exist

11. The value of the following limit:

$$\lim_{z \rightarrow \infty} \frac{iz^3 + iz - 1}{(2z + 3i)(z - i)^2}$$

- (a) $\frac{i}{2}$ (b) $-\frac{i}{2}$ (c) $\frac{1}{2}$ (d) does not exist

12. Which of the following statement is true for the complex function $f(z) = \frac{1}{z}$?

- (a) $f(z)$ is analytic everywhere (b) $f(z)$ is analytic everywhere except $z = 0$
(c) $f(z)$ is analytic nowhere (d) $f(z)$ is analytic everywhere except $z = 1$

13. Which of the following are not a complex analytic function of z ?

- (a) $f(z) = (x^2 - y^2 + 2ixy)^7 (x + iy)^{17}$ (b) $f(z) = (x^2 - y^2 + 2ixy)^{12} (x - iy)^4$
(c) $f(z) = (x + iy - 5)^{13}$ (d) $f(z) = (2x + iy - 5)^{19}$

14. Given: $f(z) = x^2 + Py^2 - 2xy + i(Qx^2 - y^2 + 2xy)$ is analytic in nature. The value of P and Q will be

- (a) $P = 1, Q = 1$, (b) $P = -1, Q = 1$, (c) $P = 1, Q = -1$, (d) $P = -1, Q = -1$

15. The value of m, n such that $f(x, y) = x^2 + mxy + ny^2$ will be a harmonic function, will be

- (a) $n = -1, m = \text{any value}$ (b) $n = 1, m = \text{any value}$
(c) $n = 0, m = -1$ (d) $n = 0, m = 1$

16. The harmonic conjugate function of $u(x, y) = 2x(1 - y)$ corresponding to a complex analytic function

$\omega = u(x, y) + iv(x, y)$ is given $v(x, y) = \alpha x^2 + \beta y + \gamma y^2$ (Taking the integration constant to be zero).

Which of the following statement is true?

- (a) $\alpha - \gamma = \beta$ (b) $\alpha + \gamma + \beta = 0$ (c) $\alpha + \gamma = \beta$ (d) $\alpha\gamma\beta = 1$

17. An electrostatic field in the x - y plane is given by the potential function $\phi(x, y) = 3x^2y - y^3$. The corresponding stream function will be

- (a) $-x^3 - 3x^2y$ (b) $-x^3 + 3xy^2$ (c) $-x^3 - 3xy^2$ (d) $x^3 - 3xy^2$



18. If the real part of a complex analytic function $f(z)$ is given as, $u(x, y) = e^{-2xy} \sin(x^2 - y^2)$, then $f(z)$ can be written as
 (a) $ie^{iz^2} + C$ (b) $-ie^{iz^2} + C$ (c) $-ie^{-iz^2} + C$ (d) $ie^{-iz^2} + C$
19. The value of the integral $\int_0^{1+i} (x^2 - iy) dz$ along the path $y = x^2$ is
 (a) $\frac{5}{6} + \frac{i}{6}$ (b) $\frac{5}{6} - \frac{i}{6}$ (c) $\frac{1}{6} - \frac{5i}{6}$ (d) $\frac{1}{6} + \frac{5i}{6}$
20. The Taylor series expansion of $f(z) = \cos z$ about $z = \frac{\pi}{6}$ will be
 (a) $f(z) = \frac{\sqrt{3}}{2} + \frac{1}{2}\left(z - \frac{\pi}{6}\right) + \frac{\sqrt{3}}{4}\left(z - \frac{\pi}{6}\right)^2 + \dots$
 (b) $f(z) = \frac{\sqrt{3}}{2} - \frac{1}{2}\left(z - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{4}\left(z - \frac{\pi}{6}\right)^2 + \dots$
 (c) $f(z) = \frac{\sqrt{3}}{2} - \frac{1}{2}\left(z - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{2}\left(z - \frac{\pi}{6}\right)^2 + \dots$
 (d) $f(z) = \frac{\sqrt{3}}{2} + \frac{1}{2}\left(z - \frac{\pi}{6}\right) - \frac{\sqrt{3}}{2}\left(z - \frac{\pi}{6}\right)^2 - \dots$
21. If $\sin^{-1}z$ is expanded in terms of z , it can be written as
 (a) $z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \dots$ (b) $z - \frac{z^3}{6} + \frac{3z^5}{40} - \frac{5z^7}{112} + \dots$
 (c) $z + \frac{z^3}{3} + \frac{z^5}{5} + \frac{z^7}{7} + \dots$ (d) $z + \frac{z^3}{6} + \frac{3z^5}{40} + \frac{5z^7}{112} + \dots$
22. Suppose a complex function $f(z)$ such that $f(1) = 1, f'(1) = 1, f''(1) = 1$ and all other higher derivatives of $f(z)$ at $z=1$. The value of $f(z)$ at $z = 1/2$ will be
 (a) $1/4$ (b) $1/2$ (c) $5/8$ (d) $7/8$
23. In the Laurent series expansion of $f(z) = \frac{(z^2 + 3z + 2)}{z(z+2)^3}$ about $z = -2$, the highest negative power (<0) terms of $(z+2)$ present in the expansion, will be
 (a) 0 (b) 1 (c) 2 (d) 3
24. The Laurent series expansion of the complex function

$$f(z) = \frac{z}{(z-1)(z-2)}$$

in the region $|z-1| > 1$ can be expressed as

(a) $\frac{1}{z-1} + \frac{2}{(z-1)^2} + \frac{2}{(z-1)^3} + \frac{2}{(z-1)^4} + \dots$



(b) $\frac{1}{z-1} - \frac{2}{(z-1)^2} + \frac{2}{(z-1)^3} - \frac{2}{(z-1)^4} + \dots$

(c) $\frac{1}{z-1} - \frac{2}{(z-1)^2} + \frac{3}{(z-1)^3} - \frac{4}{(z-1)^4} + \dots$

(d) $\frac{1}{z-1} - \frac{2}{(z-1)^2} + \frac{1}{(z-1)^3} - \frac{2}{(z-1)^4} + \dots$

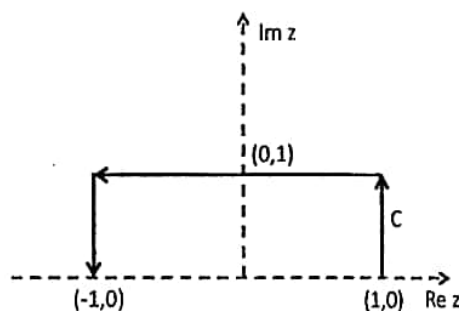
25. The function $f(z) = (z-3) \sin\left(\frac{1}{z+2}\right)$ has singularity at $z = -2$ of the following nature :
- (a) pole of order 1 (b) removable isolated singularity
(c) isolated essential singularity (d) non-isolated essential singularity
26. The function $f(z) = \frac{1}{1-e^z}$ has singularity at $z = 2\pi i$ of the following nature:
- (a) pole of order 1 (b) removable isolated singularity
(c) isolated essential singularity (d) non-isolated essential singularity
27. The function $f(z) = (z-3)^n \sin\left(\frac{1}{z-3}\right)$ has a residue of $\frac{1}{120}$ at the point $z = 3$. The value of 'n' is equal to
- (a) 2 (b) 4 (c) 5 (d) 6
28. The contribution of the point $z = \pi/2$ in evaluation of $\oint_C \frac{\tan z}{z} dz$ (where C is a circle $|z| = 2$) is equal to
- (a) 0 (b) $-4e^{i\pi/2}$ (c) $4e^{i\pi/2}$ (d) $-2/\pi$
29. The value of the integral $\oint_C \frac{z dz}{(9-z^2)(z+i)}$, where C is a circle $|z| = 2$ in the argand plane, described in the positive sense is equal to
- (a) $\pi/2$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/5$
30. The value of the integral $\oint_C \frac{3z^2 + z + 1}{(z^2 - 1)(z + 3)} dz$ around the curve $C : |z| = 2$ (where, 'C' is traversed in the clockwise direction) is equal to
- (a) $\frac{3\pi i}{4}$ (b) $-\frac{3\pi i}{4}$ (c) $\frac{\pi i}{4}$ (d) $-\frac{\pi i}{4}$
31. The value of the integral $\oint_C \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$, where 'C' is the circle $|z| = 3$ traversed in the anticlockwise direction, will be
- (a) $2\pi i$ (b) $-2\pi i$ (c) $4\pi i$ (d) $-4\pi i$
32. The value of the integral $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$ will be
- (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{6}$



33. The value of the integral $\int_0^{2\pi} e^{\cos\theta} \cos(\sin\theta - n\theta) d\theta$ will be
 (a) $\frac{2\pi}{(n+1)!}$ (b) $\frac{2\pi}{n!}$ (c) $\frac{\pi}{n!}$ (d) $\frac{\pi}{(n+1)!}$
34. The value of the integral $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$ will be
 (a) $\frac{5\pi}{12}$ (b) $\frac{5\pi}{24}$ (c) $\frac{7\pi}{12}$ (d) $\frac{7\pi}{24}$
35. The value of the integral $\int_0^{\infty} \frac{\sin mx}{x(x^2 + a^2)} dx$ will be
 (a) $\frac{\pi}{2a^2}(1 + e^{-ma})$ (b) $\frac{\pi}{2a^2}(1 - e^{-ma})$ (c) $\frac{\pi}{2a^2}(1 - e^{ma})$ (d) $\frac{\pi}{2a^2}(1 + e^{ma})$
36. The following integral $\int_0^{\infty} \frac{\cos 3x}{(x^2 + 1)(x^2 + 4)} dx$ has a value
 (a) $\frac{\pi}{2} \left[\frac{e^3}{3} - \frac{e^6}{6} \right]$ (b) $\frac{\pi}{2} \left[\frac{e^{-3}}{3} - \frac{e^{-6}}{6} \right]$ (c) $\pi \left[\frac{e^{-3}}{3} - \frac{e^{-6}}{6} \right]$ (d) $\pi \left[\frac{e^3}{3} - \frac{e^6}{6} \right]$

CSIR Previous Year Questions

37. The value of the integral $\int_C dz z^2 e^z$, where C is an open contour in the complex z-plane as shown in figure below: [CSIR June 2011]



- (a) $\frac{5}{e} + e$ (b) $e - \frac{5}{e}$ (c) $\frac{5}{e} - e$ (d) $-\frac{5}{e} - e$
38. Which of the following is an analytic function of the complex variable $z = x + iy$ in the domain $|z| < 2$? [CSIR June 2011]
- (a) $(3 + x - iy)^7$ (b) $(1 + x + iy)^4 (7 - x - iy)^3$
 (c) $(1 - 2x - iy)^4 (3 - x - iy)^3$ (d) $(x + iy - 1)^{1/2}$



39. The first few terms in the Taylor series expansion of the function $f(x) = \sin x$ around $x = \frac{\pi}{4}$ are

[CSIR Dec 2011]

- (a) $\frac{1}{\sqrt{2}} \left[1 + \left(x - \frac{\pi}{4} \right) + \frac{1}{2!} \left(x - \frac{\pi}{4} \right)^2 + \frac{1}{3!} \left(x - \frac{\pi}{4} \right)^3 + \dots \right]$
 (b) $\frac{1}{\sqrt{2}} \left[1 + \left(x - \frac{\pi}{4} \right) + \frac{1}{2!} \left(x - \frac{\pi}{4} \right)^2 - \frac{1}{3!} \left(x - \frac{\pi}{4} \right)^3 + \dots \right]$
 (c) $\left[\left(x - \frac{\pi}{4} \right) - \frac{1}{3!} \left(x - \frac{\pi}{4} \right)^3 + \dots \right]$ (d) $\frac{1}{\sqrt{2}} \left[1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots \right]$

40. The first few terms in the Laurent series for $\frac{1}{(z-1)(z-2)}$ in the region $1 \leq |z| \leq 2$ and around $z=1$ is

[CSIR June 2012]

- (a) $\frac{1}{2} \left[1 + z + z^2 + \dots \right] \left[1 + \frac{z}{2} + \frac{z^2}{4} + \frac{z^3}{8} + \dots \right]$ (b) $\frac{1}{1-z} + \frac{1}{1-z} (1-z)^2 + (1-z)^3 + \dots$
 (c) $\frac{1}{z^2} \left[1 + \frac{1}{z} + \frac{1}{z^2} + \dots \right] \left[1 + \frac{2}{z} + \frac{4}{z^2} + \dots \right]$ (d) $2(z-1) + 5(z-1)^2 + 7(z-1)^3 + \dots$

41. The value of the integral $\int_{-\infty}^{\infty} \frac{1}{t^2 + R^2} \cos\left(\frac{\pi t}{2R}\right) dt$

[CSIR June 2012]

- (a) $-\frac{2\pi}{R}$ (b) $-\frac{\pi}{R}$ (c) $\frac{\pi}{R}$ (d) $\frac{2\pi}{R}$

42. The Taylor series expansion of the function $\ln(\cosh x)$, where x is real, about point $x=0$ starts with the following terms:

[CSIR Dec 2012]

- (a) $-\frac{1}{2}x^2 + \frac{1}{12}x^4 + \dots$ (b) $\frac{1}{2}x^2 - \frac{1}{12}x^4 + \dots$
 (c) $-\frac{1}{2}x^2 + \frac{1}{6}x^4 + \dots$ (d) $\frac{1}{2}x^2 + \frac{1}{6}x^4 + \dots$

43. The value of the integral $\oint_C \frac{z^3}{z^2 - 5z + 6} dz$, where C is closed contour defined by the equation

$2|z| - 5 = 0$, traversed in the anti-clockwise direction, is

[CSIR Dec 2012]

- (a) $-16\pi i$ (b) $16\pi i$ (c) $8\pi i$ (d) $2\pi i$

44. With $z = x + iy$, which of the following functions $f(x, y)$ is NOT a complex analytic function of z ?

[CSIR June 2013]

- (a) $(x + iy - 8)^3 (4 + x^2 - y^2 + 2ixy)^7$ (b) $(x + iy)^7 (1 - x - iy)^3$
 (c) $(x^2 - y^2 + 2ixy - 3)^5$ (d) $(1 - x + iy)^4 (2 + x + iy)^6$



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45. Which of the following functions cannot be the real part of a complex analytic function of $z = x + iy$?

[CSIR Dec 2013]

- (a) x^2y (b) $x^2 - y^2$ (c) $x^3 - 3xy^2$ (d) $3x^2y - y - y^3$

46. Given that the integral $\int_0^\infty \frac{dx}{y^2 + x^2} = \frac{\pi}{2y}$, the value of $\int_0^\infty \frac{dx}{(y^2 + x^2)^2}$ is

[CSIR Dec 2013]

- (a) $\frac{\pi}{y^3}$ (b) $\frac{\pi}{4y^3}$ (c) $\frac{\pi}{8y^3}$ (d) $\frac{\pi}{2y^3}$

47. The function $\phi(x, y, z, t) = \cos(z - vt) + \text{Re}[\sin(x + iy)]$ satisfies the equation

[CSIR June 2014]

- (a) $\frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \phi$ (b) $\left(\frac{1}{v^2} \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2} \right) \phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \phi$
 (c) $\left(\frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} \right) \phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) \phi$ (d) $\left(\frac{\partial^2}{\partial z^2} - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right) \phi = \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) \phi$

48. If C is the contour defined by $|z| = \frac{1}{2}$, the value of the integral $\oint_C \frac{dz}{\sin^2 z}$ is

- (a) 0 (b) $2\pi i$ (c) 0 (d) πi

ANSWER KEY

- 1.(c) 2.(b) 3.(d) 4.(d) 5.(b) 6.(b) 7.(a) 8.(a)(d) 9.(a) 10.(d)
 11.(a) 12.(b) 13.(b)(d) 14.(b) 15.(a) 16.(a) 17.(b) 18.(b) 19.(a) 20.(b)
 21.(d) 22.(c) 23.(c) 24.(a) 25.(c) 26.(a) 27.(b) 28.(a) 29.(d) 30.(c)
 31.(c) 32.(a) 33.(b) 34.(a) 35.(b) 36.(b) 37.(c) 38.(b) 39.(b) 40.(b)
 41.() 42.() 43.(a) 44.(d) 45.(a) 46.(b) 47.(a) 48.(c)





SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

SECTION: MATHEMATICAL PHYSICS

Assignment-5: COMPLEX ANALYSIS

- If $m = \cos \theta + i \sin \theta$, then prove that $1 + m + m^2 = e^{i\theta} (1 + 2 \cos \theta)$
- If $m = e^{i\theta}$, $n = e^{i\phi}$, then show that $\frac{m-n}{m+n} = i \tan \left(\frac{\theta - \phi}{2} \right)$
- Find the complex numbers 'z' such that $\arg(z+1) = \frac{\pi}{6}$ and $\arg(z-1) = 2\frac{\pi}{3}$ [Ans. $z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$]
- Find the area of the triangle enclosed by the vectors $z, iz, z-iz$. [Ans. $\frac{1}{2}|z|^2$]
- If $2 \cos \theta = m + \frac{1}{m}$ and $2 \cos \phi = n + \frac{1}{n}$, then prove that $m^p n^q + \frac{1}{m^p n^q} = 2 \cos(p\theta + q\phi)$
- If 'n' be a positive integer, prove that $(1+i)^n + (1-i)^n = 2^{\frac{n+1}{2}} \cos \frac{n\pi}{4}$
- Check whether the following complex functions are analytic or not:
 (i) $e^x (\cos y + i \sin y)$ (ii) $\sin z$ (iii) $e^{\sin z}$ (iv) $r^n e^{in\theta}$
- Determine the imaginary or real part of the complex analytic function whose real or imaginary part are the following:
 (i) $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ (ii) $v(x, y) = 6xy - 5x + 3$
 (iii) $v(r, \theta) = r^2 \cos 2\theta - r \cos \theta + 2$ (iv) $u(r, \theta) = -r^3 \sin 3\theta$
 [Ans. (i) $3x^2 y + 6xy - y^3 + C$, (ii) $3x^2 - 3y^2 + 5y + C$, (iii) $-r^2 \sin 2\theta + r \sin \theta + C$, (iv) $r^3 \cos 3\theta + C$]
- Use Milne Thomson Method, find the complex analytic function $f(z)$ for which either the real part or the imaginary part is given as following:
 (i) $v = e^x (x \cos y - y \sin y)$ (ii) $u - v = (x - y)(x^2 + 4xy + y^2)$ (iii) $v = \tan^{-1} \left(\frac{y}{x} \right)$
 [Ans. (i) $ize^z + C$, (ii) $-iz^3 + C$, (iii) $\ln z + C$]
- Show that, the function $u = \sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 4xy$ is harmonic in nature and if the above function is the real part of a complex analytic function, then determine the corresponding analytic function.
 [Ans. $(1-2i)(\sin z + z^2) + C$]



11. If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u - v = \frac{e^y - \cos x + \sin x}{\cosh y - \cos x}$, find $f(z)$ subject to the condition $f\left(\frac{\pi}{2}\right) = \frac{3-i}{2}$. [Ans. $f(z) = \cot\left(\frac{z}{2}\right) + \frac{1-i}{2}$]
12. Find the first three terms of the Taylor's series expansion of $f(z) = \frac{z}{(z-3)(z-4)}$ about $z = 2$. Find the region of convergence.
13. Find the first three terms of the Taylor's series expansion of $f(z) = \frac{2z^3 + 1}{z^2 + z}$ about $z = 1$.
14. Expand $f(z) = \tan^{-1} z$ in powers of z . ($|z| < 1$).
15. Expand the following function in Laurent series:
- (i) $f(z) = \frac{1}{z(z+2)^3}$ about the pole $z = -2$.
- (ii) $f(z) = \frac{1}{z^2 - 3z + 2}$ in the region (i) $|z| < 1$, (ii) $|z| > 3$
16. Discuss the nature of singularity regarding the following complex functions:
- (i) $f(z) = \frac{1}{\cos z - \sin z}$ (ii) $f(z) = \frac{\cot \pi z}{(z-a)^2}$ at $z = a$, $z = \infty$ (iii) $f(z) = \tan\left(\frac{1}{z}\right)$ at $z = 0$
- (iv) $f(z) = \sin\left(\frac{1}{1-z}\right)$ (v) $f(z) = \frac{1-e^z}{1+e^z}$ at $z = \infty$ (vi) $f(z) = \frac{z}{1+z^4}$
- (vii) $f(z) = \frac{\sin z}{(z-\pi)^2}$
17. Determine the poles and corresponding residues of following complex functions:
- (i) $f(z) = \frac{1-2z}{z(z-1)(z-2)}$ (ii) $f(z) = \frac{z^2}{z^2 + a^2}$ (iii) $f(z) = \frac{ze^{iz}}{z^4 + a^4}$
- (iv) $f(z) = \frac{z^2}{(z+1)^2(z-2)}$ (v) $f(z) = \frac{z}{\sin z}$ (vi) $f(z) = \frac{e^z}{z^2(z^2+9)}$ at $(0, -3)$
- (vii) $f(z) = \frac{2z+1}{z^2 - z - 2}$ (viii) $f(z) = z^2 e^{1/z}$ (ix) $f(z) = z^2 \sin\left(\frac{1}{z}\right)$
18. Evaluate the following integral:
- (i) $\oint_C \frac{z}{z^2 - 3z + 2} dz$; C : circle of $|z-2| = \frac{1}{2}$ (ii) $\oint_C \frac{dz}{z(z+\pi i)}$; C : circle of $|z+3i| = 1$
- (iii) $\oint_C \frac{\cosh(\pi z)}{z(z^2+1)} dz$; C : circle of $|z| = 2$ (iv) $\oint_C \frac{4-3z}{z(z-1)(z-2)} dz$; C : circle of $|z| = \frac{3}{2}$



$$(v) \oint_C \frac{1}{\sinh z} dz; C : \text{circle of } |z| = 4$$

$$(vi) \oint_C \frac{e^{2z} + z^2}{(z-1)^5} dz; C : \text{circle of } |z| = 2$$

$$(vii) \oint_C z^4 e^{1/z} dz; C : \text{circle of } |z| = 1$$

$$(viii) \oint_C \frac{\sin z}{z^6} dz; C : \text{circle of } |z| = 2$$

$$[Ans. (i) 4\pi i, (ii) -2, (iii) 4\pi i, (iv) 2\pi i, (v) -2\pi i, (vi) \frac{4\pi i e^2}{3}, (vii) \frac{\pi i}{60}, (viii) \frac{\pi i}{60}]$$

19. Evaluate the following integral:

$$(i) \int_0^{2\pi} \frac{\sin^2 \theta}{5 - 4 \cos \theta} d\theta$$

$$(ii) \int_0^{2\pi} \frac{1 + 2 \cos \theta}{5 + 4 \cos \theta} d\theta$$

$$(iii) \int_0^{2\pi} \frac{d\theta}{1 - 2m \cos \theta + m^2}, (m^2 < 1)$$

$$(iv) \int_0^{2\pi} \frac{\sin^2 \theta - 2 \cos \theta}{2 + \cos \theta} d\theta$$

$$[Ans. (i) \frac{\pi}{4}, (ii) 0, (iii) \frac{2\pi}{1-m^2}, (iv) \frac{2\pi}{\sqrt{3}}]$$

20. Evaluate the following integral:

$$(i) \int_{-\infty}^{\infty} \frac{1}{(1+x^2)^3} dx$$

$$(ii) \int_0^{\infty} \frac{dx}{x^4 + a^4}$$

$$(iii) \int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)(x^2+4)} dx$$

$$(iv) \int_0^{\infty} \frac{\cos mx}{(x^2 + a^2)} dx; (a > 0, m > 0)$$

$$(v) \int_0^{\infty} \frac{\cos 2x}{(x^2 + a^2)(x^2 + b^2)} dx (a > b > 0)$$

$$(vi) \int_0^{\infty} \frac{x^3 \sin mx}{(x^4 + a^4)} dx; (a > 0, m > 0)$$

$$[Ans. (i) \frac{3\pi}{8}, (ii) \frac{\pi}{\sqrt{2}a^3}, (iii) \frac{\pi}{3}, (iv) \frac{\pi}{2a} e^{-ma}, (v) \frac{\pi}{2(a^2 - b^2)} \left(\frac{e^{-2b}}{b} - \frac{e^{-2a}}{a} \right), (vi) \frac{\pi}{2} \exp\left(-\frac{ma}{\sqrt{2}}\right) \cos\left(\frac{ma}{\sqrt{2}}\right)]$$





CSIR-UGC-NET/JRF/GATE-PHYSICAL SCIENCES

SECTION: MATHEMATICAL PHYSICS

Assignment-8: PROBABILITY

1. A triangle is formed by joining three points of a regular hexagon. What is the probability that the triangle will be equilateral?
(a) $1/6$ (b) $1/10$ (c) $1/12$ (d) $1/20$
2. Two dice are thrown once the probability of getting a sum 9 is given by:
(a) $\frac{1}{12}$ (b) $\frac{1}{18}$ (c) $\frac{1}{6}$ (d) None of these
3. Let $P \begin{bmatrix} 1 & a \\ b & c \end{bmatrix}$, where a, b, c are chosen randomly from the set $\{1, 2, 3, 4, 5\}$. The probability that P is singular is
(a) $\frac{2}{25}$ (b) $\frac{1}{11}$ (c) $\frac{2}{21}$ (d) $\frac{1}{5}$
4. What is the probability that a leap year, selected at random, will contain 53 Mondays?
(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{1}{5}$ (d) $\frac{2}{5}$
5. What is the probability that the numbers formed between 60000 and 70000 using the digits 1,2,3,4,5,6,7,8,9 should end with '7'? (no digit is repeated)
(a) $\frac{2}{5}$ (b) $\frac{2}{7}$ (c) $\frac{1}{8}$ (d) $\frac{1}{11}$
6. What is probability that the words formed by the letters of the word "STRANGE" can be made so that the vowels should be in even places?
(a) $\frac{1}{7}$ (b) $\frac{2}{9}$ (c) $\frac{3}{11}$ (d) $\frac{4}{7}$
7. What is the probability that a 6-digit telephone number formed using digits 0 to 9, should start with 2 and end with 8? (no digits is repeated)
(a) $\frac{1}{32}$ (b) $\frac{1}{45}$ (c) $\frac{1}{90}$ (d) $\frac{2}{35}$
8. A bag contains 10 black and 15 red balls. Two balls are drawn from the bag at random. The probability that two balls drawn from the bag will be of same colour?
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{5}$



9. Consider the experiment of throwing two fair dice. What is the probability that the sum of the numbers obtained in these dice is even ?
 (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$
10. A bag contains 6 red and 4 green balls. A fair dice is rolled and a number of balls equal to that appearing on the dice is chosen from the urn at random. The probability that all the balls selected are red is.
 (a) $\frac{1}{3}$ (b) $\frac{3}{10}$ (c) $\frac{1}{8}$ (d) $\frac{68}{315}$
11. A six faced die is a biased one. it is thrice more likely to show an odd number than to show an even number. It is thrown twice. The probability that the sum of the numbers in the two throws is even, is.
 (a) $\frac{4}{8}$ (b) $\frac{5}{8}$ (c) $\frac{6}{8}$ (d) $\frac{7}{8}$
12. Two balls are drawn at random from a bag containing 6 white, 4 red and 5 black balls. The probability that both these balls are black, is:
 (a) $\frac{1}{21}$ (b) $\frac{2}{15}$ (c) $\frac{2}{21}$ (d) $\frac{2}{35}$
13. A problem is given to three students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. The probability that this problem will be solved is:
 (a) $\frac{1}{24}$ (b) $\frac{1}{6}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$
14. 6 boys and 6 girls sit in a row randomly. The probability that all the girls sit together is:
 (a) $\frac{1}{132}$ (b) $\frac{1}{44}$ (c) $\frac{5}{132}$ (d) $\frac{7}{132}$
15. Different words are written with the letters of PEACE. The probability that both E's come together is:
 (a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$
16. The probability of randomly choosing 3 defectless bulbs from 15 electric bulbs of which 5 bulbs are defective is:
 (a) $\frac{3}{10}$ (b) $\frac{7}{10}$ (c) $\frac{24}{91}$ (d) $\frac{67}{91}$
17. The chance of throwing 12 in a single throw with three dice is
 (a) $\frac{12}{216}$ (b) $\frac{21}{216}$ (c) $\frac{15}{216}$ (d) $\frac{25}{216}$
18. The probability that a number in $\{1, 2, \dots, 1001\}$ is divisible by 7 or 11 or both, is
 (a) $\frac{143}{1001}$ (b) $\frac{221}{1001}$ (c) $\frac{247}{1001}$ (d) $\frac{91}{1001}$
19. A point P is chosen at random on a line AB of length $2l$. The probability $P[(AP \times BP) > l^2 / 2]$ is
 (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\left(1 - \frac{1}{\sqrt{2}}\right)l$ (d) $\frac{1}{\sqrt{2}}l$



20. A number is chosen from each of the two sets $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. If P_1 denotes the probability that the sum of the two numbers be 10 and P_2 the probability that their sum be 8, then $(P_1 + P_2)$ is
 (a) $7/729$ (b) $137/729$ (c) $16/81$ (d) $137/81$
21. The probability that three men hit a target are $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}$ respectively. Each shoots once the target. What is the probability that exactly one of them hits the target?
 (a) $\frac{23}{96}$ (b) $\frac{42}{127}$ (c) $\frac{19}{48}$ (d) $\frac{31}{72}$
22. From a pack of 52 cards two are drawn at random. What is the probability that one is a king and the other a queen?
 (a) $\frac{5}{629}$ (b) $\frac{8}{663}$ (c) $\frac{10}{687}$ (d) None of the above
23. The probability that the sum of two numbers x and y randomly chosen in the interval $(0, 1)$ greater than 1 while the sum of the squares less than 1 is equal to
 (a) $\frac{2}{\pi}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6} - \frac{1}{2}$ (d) $\frac{\pi}{4} - \frac{1}{2}$
24. The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 3 is
 (a) $16/25$ (b) $9/10$ (c) $(9/10)^2$ (d) $18/25$
25. At a railway station a passenger leaves his luggage in a locker which is opened by dialling a three-digit code. The passenger chooses the code, closes the locker and leaves for the town. A stranger who does not know the code tries to open the locker by dialling three digits at random without repeating the unsuccessful codes. The probability that the stranger does not have to perform more than 100 trials for a success is
 (a) $\frac{9}{10}$ (b) $\frac{999}{1000}$ (c) $\frac{1}{1000}$ (d) $\frac{1}{10}$
26. A cow is tied with a pole by a 100 meter long rope. What is the probability that at some point of time the cow is at least 60 meters away from the pole?
 (a) $\frac{9}{25}$ (b) $\frac{13}{25}$ (c) $\frac{16}{25}$ (d) $\frac{18}{25}$
27. A and B play a game of dice. A throws the die first. The person who first gets a 6 is the winner. What is the probability that A wins?
 (a) $\frac{6}{11}$ (b) $\frac{1}{2}$ (c) $\frac{5}{6}$ (d) $\frac{1}{6}$
28. Two teams A and B play a series of four matches. If the probability that team A wins a match is $\frac{2}{3}$, then the probability that team A wins three matches, loses one and the third win occurs in the fourth match is
 (a) $\frac{8}{27}$ (b) $\frac{16}{27}$ (c) $\frac{8}{81}$ (d) $\frac{32}{81}$



29. The probabilities that a husband and wife will be alive 20 years from now are given by 0.8 and 0.9 respectively. What is the probability that in 20 years at least one, will be alive?
 (a) 0.98 (b) 0.02 (c) 0.72 (d) 0.28
30. What is the probability that an urn containing 5 balls contains only white balls if the first two balls drawn from it was found to be white
 (a) $1/20$ (b) $1/10$ (c) $1/2$ (d) $1/4$
31. If the probabilities that A and B will die within a year are p and q respectively, then the probability that only one of them will be alive at the end of the year is
 (a) pq (b) $p(1 - q)$ (c) $q(1 - p)$ (d) $p + q - 2pq$
32. If A and B are two events such that $P(A) = p$, $P(B) = q$, $P(A \cap B) = r$ then the probability that A occurs but B does not occur is
 (a) $p - r$ (b) $p - q$ (c) $p - qr$ (d) $p - pq$
33. A box contains n enumerated articles. All the articles are taken out one by one at random. The probability that the numbers of the selected articles are in the sequence 1, 2, ..., n is
 (a) $\frac{1}{n!}$ (b) $\frac{n}{n!}$ (c) $\frac{1}{n(n!)}$ (d) none of the above
34. An anti-aircraft gun can take a maximum of four shots at a plane moving away from it. The probability of hitting the plane in the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that the gun hits the plane?
 (a) 0.7325 (b) 0.7286 (c) 0.6976 (d) 0.6574
35. For n independent events $A_1, A_2, A_3, \dots, A_n$, let $P(A_i) = \frac{1}{i+1}$, $i = 1, 2, \dots, n$. Then the probability that none of the events will occur is
 (a) $n/(n+1)$ (b) $n - 1/(n+1)$ (c) $1/(n+1)$ (d) $1/n$
36. Let A and B be two events with $P(A) = 1/2$, $P(B) = 1/3$ and $P(A \cap B) = 1/4$. What is $P(A \cup B)$?
 (a) $3/7$ (b) $4/7$ (c) $7/12$ (d) $9/12$
37. Two events A and B have probabilities 0.25 and 0.50 respectively. The probability that A and B occur is 0.14. The probability that neither A nor B occur will be
 (a) 0.61 (b) 0.39 (c) 0.86 (d) 0.75
38. If $P(A^c \cap B^c) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{2}$ and $P(B) = 2P(A)$, the $P(A)$ will be
 (a) $4/7$ (b) $11/18$ (c) $7/18$ (d) $13/18$
39. A purse contains 2 silver and 4 copper coins. A second purse contains 4 silver and 3 copper coins. If a coin is pulled out at random from one of the two purses, then the probability that it will be silver coin will be
 (a) $2/7$ (b) $4/7$ (c) $19/42$ (d) $23/42$
40. Let A, B and C be independent and mutually exclusive events with probability of occurrences $\frac{1-3p}{2}$, $\frac{1+4p}{3}$, $\frac{1+p}{6}$, respectively, then p lies in



- (a) $[-1/4, 5/6]$ (b) $[-1/4, 1/3]$ (c) $[-1/4, 1/2]$ (d) $[-1/2, 1/3]$

41. Three persons play a game by tossing a fair coin each independently. The game ends in a trial if all of them get the same outcome in that trial, otherwise they continue to the next trial. What is the probability that the game ends in an even number of trials?
- (a) $2/7$ (b) $3/7$ (c) $1/2$ (d) $4/7$
42. In a certain office, $1/3$ of the workers are women, $1/2$ of the women are married and $1/3$ of the married women have children. If $3/4$ of the men are married and $2/3$ of the married men have children, what part of workers are without children?
- (a) $5/18$ (b) $4/9$ (c) $11/18$ (d) $17/36$
43. A and B are independent witnesses in a case. The probability that A speaks the truth is 'x' and that B speaks the truth is 'y'. If A and B agree on a certain statement, the probability that the statement is true is
- (a) $\frac{xy}{xy + (1-x)(1-y)}$ (b) $\frac{xy}{(1-x)(1-y)}$
- (c) $\frac{(1-x)(1-y)}{xy + (1-x)(1-y)}$ (d) $\frac{(1-x)(1-y)}{xy}$
44. A bag contains 4 white and 3 black balls and a second bag contains 3 white and 3 black balls. If a ball is drawn from each of the bags, then the probability that both are of same colour is :
- (a) $\frac{3}{14}$ (b) $\frac{1}{2}$ (c) $\frac{3}{13}$ (d) $\frac{1}{3}$
45. Suppose there are two bags with first bag contains 3 white and 2 black balls, second bag contains 2 white and 4 black balls. one ball is transferred from bag I to bag II. and then a ball is drawn from the latter. It happens to be white. What is the probability that the transferred ball is white?
- (a) $\frac{9}{13}$ (b) $\frac{7}{13}$ (c) $\frac{6}{13}$ (d) $\frac{11}{13}$
46. Three urns are given each containing red and white balls. Urn I contains 6 red and 4 white balls, Urn II contains 2 red and 6 white balls and Urn III 1 red and 4 white balls. An urn is selected at random and a ball is drawn. If the selected ball is red, what is the probability that it is from the first urn?
- (a) $\frac{2}{5}$ (b) $\frac{2}{7}$ (c) $\frac{4}{7}$ (d) $\frac{4}{9}$
47. The probability of getting a defective floppy in three boxes A, B and C are $1/3$, $1/6$, and $3/4$, respectively. A box is selected randomly and a floppy is drawn from it. The probability that the floppy is defective and is drawn from box A is
- (a) $4/15$ (b) $12/15$ (c) $2/15$ (d) $3/5$
48. A box contains 99 fair coins and 1 double-headed coin. One coin is chosen from the box at random. The coin is tossed 3 times and we get heads all the time. What is the probability that the chosen coin will be double-headed?
- (a) 0.075 (b) 0.18 (c) 0.26 (d) 0.31



Suppose, 'x' is a continuous random variable with probability density function given by,

$$\begin{aligned} f(x) &= kx; (0 \leq x < 2) \\ &= 2k; (2 \leq x < 4) \\ &= -kx + 6k; (4 \leq x < 6) \end{aligned}$$

49. The value of 'k' will be
 (a) 1/8 (b) 1/6 (c) 1/2 (d) 4/3
50. The mean value of the random variable 'x' will be
 (a) 3/2 (b) 3 (c) 5/3 (d) 7/4
51. The probability density function of a continuous random variable is given as,

$$f(x) = Ke^{-|x|} \quad (-\infty < x < \infty)$$

- (a) Find the value of 'K'.
 (b) Find the mean and variance of the distribution.
 (c) Find the probability that the variate lies between 0 and 4.

[Ans. (a) 1/2, (b) 0, 2, (c) $\frac{1}{2}(1 - e^{-4})$]

52. A continuous random variable has the following probability density function:

$$f(x) = a + bx \quad \text{for } 0 \leq x \leq 1$$

If the mean of the distribution is 1/2, the value of a and b will be

- (a) $a = 1, b = 1$ (b) $a = 0, b = 1$ (c) $a = 1, b = 0$ (d) $a = 1, b = -1$

Common Data Questions: Q.48 and Q.49

The probability density function of the random variable 'x' is given as

x:	0	1	2	3	4	5	6
P(x):	k	3k	5k	7k	9k	11k	13k

53. The probability that the value of 'x' will be less than 4, will be
 (a) 9/49 (b) 16/49 (c) 24/49 (d) 25/49
54. The probability that the value of 'x' will be atleast '5', will be
 (a) 9/49 (b) 16/49 (c) 24/49 (d) 25/49

Common Data Questions: Q.50 and Q.51

Suppose an probability distribution is defined as

$$f(x) = \frac{1}{n} \quad \text{for } x = 1, 2, 3, \dots, n$$

55. The mean of the distribution will be
 (a) $\frac{n+1}{2}$ (b) $\frac{n}{2}$ (c) $\frac{n-1}{2}$ (d) $\frac{n}{2} - 1$



56. The variance of the distribution will be

- (a) $\frac{n^2 + 1}{12}$ (b) $\frac{n^2 - 1}{12}$ (c) $\frac{n^2 - 1}{6}$ (d) $\frac{n^2 + 1}{6}$

57. A die is tossed once. If the random variable X is defined as

$$x = \begin{cases} 1, & \text{if the die results in an even number} \\ 0, & \text{if the die results in an odd number} \end{cases}$$

The mean of the probability distribution will be

- (a) $1/3$ (b) $1/4$ (c) $1/2$ (d) $2/3$

58. An unbiased coin is tossed 6 times. What is the probability of obtaining 4 heads?

- (a) $3/16$ (b) $15/64$ (c) $15/32$ (d) $21/64$

59. The overall percentage of failure in an examination is 20 percent. If 7 candidates appear in the examination, what is the probability that atleast 6 pass the examination?

- (a) $\frac{4^6 \cdot 11}{5^7}$ (b) $\frac{4^7 \cdot 11}{5^7}$ (c) $\frac{4^7 \cdot 11}{5^6}$ (d) $\frac{4^6 \cdot 11}{5^6}$

60. A die is thrown 9 times. What is the probability of getting '5' exactly 3 times?

- (a) $\frac{5^6 \cdot 14}{6^9}$ (b) $\frac{5^6 \cdot 14}{6^8}$ (c) $\frac{5^6 \cdot 84}{6^8}$ (d) $\frac{5^3 \cdot 84}{6^8}$

61. The chance that out of 10 telephone lines, one is busy at a time is 0.8. What will be the most probable number of busy lines?

- (a) 2 (b) 5 (c) 8 (d) 10

62. Three unbiased coins are tossed together and this is repeated five times. What is the probability of getting atleast one head each time?

- (a) $\frac{1}{8}$ (b) $\left(\frac{1}{8}\right)^5$ (c) $\frac{7}{8}$ (d) $\left(\frac{7}{8}\right)^5$

63. Which of the following is NOT an assumption of the Binomial distribution?

- (a) All trials must be identical.
(b) All trials must be independent.
(c) Each trial must be classified as a success or a failure.
(d) The probability of success is equal to 0.5 in all trials.

64. In a triangle test, a tester is presented with three food samples, two of which are alike, and is asked to pick out the odd one by testing. If a tester has no well developed sense and can pick the odd one only, by chance, what is the probability that in five trials he will make four or more correct decisions?

- ✓(a) $11/243$ (b) $1/243$ (c) $10/243$ (d) $233/243$

65. The probability that a certain machine will produce a defective item is $1/4$. If a random sample of 6 items is taken from the output of this machine, what is the probability that there will be 5 or more defectives in the sample?

- (a) $3/4096$ (b) $4/4096$ (c) $18/4096$ (d) $19/4096$



66. Suppose 60% of a herd of cattle is infected with a particular disease. Let Y = the number of non-diseased cattle in a sample of size 5. The distribution of Y is
 (a) binomial with $n = 5$ and $p = 0.6$
 (b) binomial with $n = 5$ and $p = 0.4$
 (c) binomial with $n = 5$ and $p = 0.5$
 (d) none of these
67. If the probability of unfavorable reaction from a certain injection is 0.001, what is the probability that out of 2000 individuals, more than 2 will suffer from unfavorable reaction?
 (a) $\frac{5}{e^2}$ (b) $1 - \frac{5}{e^2}$ (c) $1 - \frac{3}{e^2}$ (d) $\frac{3}{e^2}$
68. The chance of a coal miner being killed in a mine accident is $1/1400$ during a calendar year. What is probability that a mine with 3500 employers will have at least one accident in a year?
 (Given: $e^{-5/2} = 0.082$)
 (a) 0.164 (b) 0.492 (c) 0.618 (d) 0.918
69. There are 20 buses passing a bus-stop randomly within an hour. What is the probability that there will be no buses within 5 minutes? (Given: $e^{-5/3} = 0.19$)
 (a) 0.19 (b) 0.23 (c) 0.38 (d) 0.52
70. A card is drawn from a well-shuffled pack of 52 cards. A sequence of 156 trials is made. What is the probability of getting a queen of clubs at least once?
 (a) $\frac{1}{e^3}$ (b) $1 - \frac{1}{e^3}$ (c) $1 - \frac{1}{e^{52}}$ (d) $\frac{1}{e^{52}}$
71. Suppose flaws (cracks, chips, specks, etc.) occur on the surface of glass with density of 3 per square metre. What is the probability of there being exactly 4 flaws on a sheet of glass of area 0.5 square metre?
 (Given: $e^{-3/2} = 0.223$)
 (a) 0.047 (b) 0.168 (c) 0.815 (d) 0.647
72. In a certain communications system, there is an average of 1 transmission error per 10 seconds. What is the probability of more than 1 error in a communication one-half minute in duration? (Given: $e^{-3} = 0.05$)
 (a) 0.2 (b) 0.32 (c) 0.56 (d) 0.8
73. Students of a class were given an aptitude test. Their marks were found to be normally distributed with mean 60 and standard deviation 5. What percentage of students scored more than 60 marks?
 (a) 0.25 (b) 0.5 (c) 0.6 (d) 0.75
74. A manufacturer produces airmail envelopes whose weight is normally distributed with a mean 1.95 gm and standard deviation 0.05 gm. The envelopes are sold in lots of 1000. How many envelopes in a lot will heavier than 2 gm?
 (a) 78 (b) 142 (c) 155 (d) 221



75. In a test on 2000 electric bulbs, it was found that the life of a particular bulb was normally distributed with an average life of 2040 hours and standard deviation of 60 hours. How many bulbs likely to burn for more than 2160 hours?
- (a) 45 (b) 65 (c) 84 (d) 105

CSIR Previous Year Questions

76. An unbiased dice is thrown three times successively. The probability that the number of dots on the uppermost surface add up to 16 is [CSIR Dec 2011]
- (a) $1/16$ (b) $1/36$ (c) $1/108$ (d) $1/216$
77. A ball is picked at random from one of the two boxes that contain 2 black and 3 white and 3 black and 4 white balls respectively. What is the probability that it is white? [CSIR June 2012]
- (a) $34/70$ (b) $41/70$ (c) $36/70$ (d) $29/70$
78. In a series of five cricket matches, one of the captains calls 'heads' every time when the toss is taken. The probability that he will win 3 times and lose 2 times is [CSIR Dec 2012]
- (a) $1/8$ (b) $5/8$ (c) $3/16$ (d) $5/16$
79. Consider three particles A, B and C, each with attribute S that can take two values ± 1 . Let $S_A = 1, S_B = 1$ and $S_C = -1$ at a given instant. In the next instant, each S value can change to $-S$ with probability $1/3$. The probability that $S_A + S_B + S_C$ remains unchanged is [CSIR June 2013]
- (a) $2/3$ (b) $1/3$ (c) $2/9$ (d) $4/9$
80. A loaded dice has the probabilities $\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21}$ of turning up 1, 2, 3, 4, 5 and 6 respectively. If it is thrown twice, what is the probability that the sum of the numbers that turn up is even? [CSIR Dec 2013]
- (a) $144/441$ (b) $225/441$ (c) $221/441$ (d) $220/441$
81. A child makes a random walk on a square lattice of lattice constant 'a', taking a step in the north, east, south or west directions with probabilities 0.255, 0.255, 0.245, 0.245 respectively. After a large number of steps N, the expected position of the child w.r.t the starting point is at a distance [CSIR Dec 2013]
- (a) $\sqrt{2} \times 10^{-2} Na$ in the north-east direction (b) $\sqrt{2N} \times 10^{-2} a$ in the north-east direction
- (c) $2\sqrt{2} \times 10^{-2} Na$ in the south-east direction (d) 0
82. In one dimension, a random walker takes a step with equal probability to the left or right. What is the probability that the walker returns to the starting point after 4 steps? [CSIR June 2014]
- (a) $3/8$ (b) $5/16$ (c) $1/4$ (d) $1/16$





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CSIR-UGC-NET/JRF/GATE-PHYSICAL SCIENCES

SECTION: MATHEMATICAL PHYSICS

Assignment-8: PROBABILITY

ANSWER KEY

1.(b)	2.(d)	3.(a)	4.(b)	5.(c)	6.(a)
7.(c)	8.(b)	9.(a)	10.(d)	11.(b)	12.(c)
13.(d)	14.(a)	15.(b)	16.(c)	17.(d)	18.(b)
19.(b)	20.(c)	21.(d)	22.(b)	23.(d)	24.(d)
25.(d)	26.(c)	27.(a)	28.(a)	29.(a)	30.(c)
31.(d)	32.(a)	33.(a)	34.(c)	35.(c)	36.(c)
37.(b)	38.(c)	39.(c)	40.(b)	41.(b)	42.(c)
43.(a)	44.(b)	45.(a)	46.(c)	47.(a)	48.(a)
49.(a)	50.(b)	52.(c)	53.(b)	54.(c)	55.(a)
56.(b)	57.(c)	58.(b)	59.(a)	60.(b)	61.(c)
62.(d)	63.(d)	64.(a)	65.(d)	66.(b)	67.(b)
68.(d)	69.(a)	70.(b)	71.()	72.()	73.(b)
74.(c)	75.(a)	76.(b)	77.(b)	78.(d)	79.(d)
80.(b)	81.(a)	82.(a)			



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SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

SECTION: MATHEMATICAL PHYSICS

Assignment-9: FOURIER SERIES

- Which of the following is NOT a periodic function of θ ?
 (a) $\sin \theta + \pi \cos \theta$ (b) $\sin(\pi \theta) + \cos(\pi \theta)$
 (c) $\sin \theta + \cos(\pi \theta)$ (d) $\sin \theta + \cos(\theta + \pi)$
- In the fourier series expansion of $y = (x^2 - 1)^{1/2}$ ($-\pi \leq x \leq \pi$) the coefficient a_1, b_1 will be
 (a) $\frac{1}{\pi}, 1$ (b) $1, -\frac{1}{\pi}$ (c) $2, \frac{\pi^2}{2}$ (d) Unable to find
- In the Fourier series expansion of $y = \frac{1}{x^2(x-2)}$ ($-2 \leq x \leq 2$) the co-efficient a_0 will be
 (a) Unable to find (b) 0 (c) $\frac{\pi^2}{3}$ (d) $\frac{2}{3}$

- Suppose $f(x)$ is defined as following:

$$f(x) = \begin{cases} 1 & -1 \leq x \leq 0 \\ -2 & 0 \leq x \leq 1 \end{cases}$$

then a_0, a_3, b_3 in the fourier series expansion of $f(x)$ are

- Suppose $f(x)$ is defined as following:

$$f(x) = \begin{cases} -k; & -\pi < x < 0 \\ k; & 0 < x < \pi \end{cases}$$

then co-efficient b_2, b_3 in the fourier series expansion will be

- Suppose $f(x)$ is defined as following:

$$f(x) = \begin{cases} 0; & -\pi < x < 0 \\ 2\pi; & 0 < x < \pi \end{cases}$$

In the fourier series expansion, the coefficient of $\cos 2x$ is:

- $\frac{2}{3\pi}$
- $\frac{1}{\pi}$
- 0
- $\frac{2}{3\pi}$

7. Suppose $f(x)$ is defined as following:

$$f(x) = \begin{cases} 0 & -\pi < x \leq 0 \\ \frac{\pi x}{4} & 0 \leq x < \pi \end{cases}$$

then the co-efficient a_n will be

- (a) $\frac{(-1)^n}{4n^2}$ (b) $\frac{(-1)^n - 1}{4n^2}$ (c) $\frac{(-1)^n - 1}{4n}$ (d) $\frac{(-1)^n}{4n}$

8. In the fourier expansion for $f(x) = x \cos x$ ($-\pi < x < \pi$). Which of the following statement is true?

- (a) $b_n = 0$ for all n (b) $b_n = \frac{2n}{n^2 - 1}$ for all even n
 (c) $b_n = -\frac{2n}{n^2 - 1}$ for all even n (d) $b_n = -\frac{2n}{n^2 - 1}$

9. In the fourier series expansion of $f(x) = |\sin x|$, which of the following co-efficients are non-zero?

- ($-\pi \leq x \leq \pi$)
 (a) a_n for odd n (b) a_n for even n (c) b_n for odd n (d) b_n for even n .

10. If $f(x) = x - x^2$ from $x = -\pi$ to π is expanded into

$$f(x) = -\frac{\pi^2}{3} + \sum \frac{4}{n^2} (-1)^{n+1} \cos nx + \sum \frac{2}{n} (-1)^{n+1} \sin nx$$

then $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ is equal to

- (a) $\frac{\pi^2}{12}$ (b) $\frac{-\pi^2}{12}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{-\pi^2}{4}$

11. The function $f(x) = x^2$, $0 < x < 2\pi$ has a Fourier series representation

$$x^2 = \frac{4\pi^2}{3} + \sum_{n=1}^{\infty} \left(\frac{4}{n^2} \cos nx - \frac{4\pi}{n} \sin nx \right)$$

At $x = 0$ and 2π , the series converges to $2\pi^2$. Then the value of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is:

- (a) $2\pi^2$ (b) π^2 (c) $\frac{\pi^2}{3}$ (d) $\frac{\pi^2}{6}$

12. If $f(x)$ be an even, period function of x which is continuous. Then

- (a) It can be represented by a cosine series.
 (b) It can be represented by a sine series
 (c) It cannot be represented by a Fourier series.
 (d) It needs both sine and cosine terms in its Fourier expansion.

13. $f_1(x) = \sin^2 x$ and $f_2(x) = \cos^2 x$ are expanded in their corresponding fourier series. Which of the following is correct?

(a) $a_2^{(1)} = \frac{1}{2}, b_2^{(2)} = -\frac{1}{2}$

(b) $b_2^{(1)} = \frac{1}{2}, a_2^{(2)} = -\frac{1}{2}$

(c) $a_2^{(1)} = \frac{1}{2}, a_2^{(2)} = -\frac{1}{2}$

(d) $b_2^{(1)} = \frac{1}{2}, b_2^{(2)} = -\frac{1}{2}$

14. Consider the Fourier series corresponding to the function

$$f(x) = \begin{cases} 3, & 0 < x < 5 \\ -3, & -5 < x < 0 \end{cases}$$

Then the fourier series contains.

(a) Cosine terms only

(b) Both cosine and sine terms

(c) Sine terms only

(d) Sine terms and non-zero constant terms.

15. The Fourier series expansion of the function $f(x) = x(\pi - x), 0 \leq x \leq 2\pi$, is:

$$\frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \dots \right)$$

The value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is deduced as

(a) $\frac{\pi^2}{6}$

(b) $\frac{2\pi^2}{6}$

(c) $-\frac{\pi^2}{6}$

(d) Zero

16. The Fourier coefficients a_n in the Fourier series expansion.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx, \text{ for } f(x) = \begin{cases} -1, & -\pi \leq x \leq 0 \\ +1, & 0 \leq x \leq \pi \end{cases}, \text{ are}$$

(a) $\frac{2}{n\pi} [1 - (-1)^n]$

(b) $\frac{4}{n\pi}$

(c) $\frac{2}{n\pi} [1 + (-1)^n]$

(d) 0

17. Find the fourier series expansion for

(i) Rectangular Wave: $f(x) = \begin{cases} -k & \text{for } -\pi < x < 0 \\ k & \text{for } 0 < x < \pi \end{cases}$

(ii) Half Wave Rectifier: $f(x) = \begin{cases} 0 & \text{for } -\frac{\pi}{\omega} < t < 0 \\ E \sin \omega t & \text{for } 0 < t < \frac{\pi}{\omega} \end{cases}$

(iii) Saw Tooth Wave: $f(x) = x + \pi$ for $-\pi < x < \pi$

Ans. (i) $\frac{4k}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right]$

(ii) $\frac{E}{\pi} + \frac{E}{2} \sin \omega t - \frac{2E}{\pi} \left[\frac{1}{1.3} \cos 2\omega t + \frac{1}{3.5} \cos 4\omega t + \dots \right]$

(iii) $\pi + 2 \left[\sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \dots \right]$

18. Obtain the complex form of the fourier series of the expansion

$$f(x) = 0; \quad -\pi \leq x \leq 0$$

$$= 1; \quad 0 \leq x \leq \pi$$

Ans. $\frac{1}{2} + \frac{1}{i\pi} \left[\frac{e^{ix}}{1} + \frac{e^{i3x}}{3} + \frac{e^{i5x}}{5} + \dots \right] + \frac{1}{i\pi} \left[-\frac{e^{-ix}}{1} - \frac{e^{-3ix}}{3} - \frac{e^{-5ix}}{5} + \dots \right]$

ANSWER KEY

- | | | | | | | |
|--------|--------|--------|--------|--------|--------|--------|
| 1.(c) | 2.(d) | 3.(a) | 4.(b) | 5.(c) | 6.(c) | 7.(b) |
| 8.(b) | 9.(b) | 10.(a) | 11.(d) | 12.(a) | 13.(c) | 14.(c) |
| 15.(a) | 16.(d) | | | | | |



SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

SECTION: MATHEMATICAL PHYSICS

Assignment-9: DIFFERENTIAL EQUATIONS

upto 23

- The degree and order of differential equation $\sqrt{2\left(\frac{dy}{dx}\right)^3 + 4} = \left(\frac{d^2y}{dx^2}\right)^{3/2}$ are respectively
(a) order 2, degree 3 (b) order 1, degree 3 (c) order 3, degree 2 (d) order 3, degree 1
- What is the degree of the differential equation $3y = x \frac{dy}{dx} + \sin\left(\frac{dy}{dx}\right)$?
(a) 1 (b) 2 (c) 8 (d) cannot be determined
- The half-life of a radioactive substance is the time required for one-half of that substance to decay. The amount of ^{14}C , an isotope of carbon present at a future time t (in months) is given by $A(t) = 100 \exp[-0.0338t]$. The half-life of the material in months is
(a) $\ln 2$ (b) 0.0338 (c) $\ln 2/0.0338$ (d) $2 \ln 2$
- A cup of coffee has a temperature 95°C and is in a room where the temperature is 20°C . Let, $T =$ temperature of the coffee after ' t ' minutes. Assuming Newton's law of cooling, which of the following options describes T ? (k is a positive constant)
(a) $\frac{dT}{dt} = k(T - 20)$ (b) $\frac{dT}{dt} = -k(T - 20)$ (c) $\frac{dT}{dt} = -k(T - 95)$ (d) $\frac{dT}{dt} = k(T - 95)$
- Suppose that, a tumour in a rat is approximately spherical and its rate of growth is proportional to its diameter. If the tumour has diameter 5 mm. When detected and 8 mm, after three months later, what will the diameter be after another three months?
(a) 10.1 mm (b) 12.8 mm (c) 11.6 mm (d) 13.2 mm
- A tank holds 1000 liters of water, in which 15 kg of salt is dissolved. Pure water enter the tank at the rate of 10 liters/minutes. The solution is kept throughout mixed and drained from the tank at the same rate. If ' m ' is the mass of the salt in the tank at any time ' t ', which of the following options describes the rate of change of the mass of the salt in the tank?
(a) $\frac{dm}{dt} = 15 - \frac{m}{100}$ (b) $\frac{dm}{dt} = -\frac{m}{100}$ (c) $\frac{dm}{dt} = -\frac{15}{100}$ (d) $\frac{dm}{dt} = -\frac{15 - m}{100}$
- The size of a population ' P ' is modeled by the differential equation $\frac{dP}{dt} = 1.2P\left(1 - \frac{P}{4200}\right)$
For what values of P , population is increasing?
(a) $P > 0$ (b) $0 < P < 4200$ (c) $P < 2100$ (d) Not enough information provided.



8. The differential equation of all circles passing through the origin and having their centres on the y-axis

(a) $(x^2 + y^2) dy/dx = 2xy$

☒ (b) $(x^2 - y^2) dy/dx = 2xy$

(c) $dy/dx = 2xy(x^2 + y^2)$

(d) $dy/dx = 2xy(x^2 - y^2)$

9. Classify the following differential equations:

$$e^x \frac{dy}{dx} + 3y = x^2 y$$

(a) Separable but not linear

(b) Linear but not separable

(c) Both separable and linear

(d) Neither separable nor linear.

10. Classify the following differential equation:

$$\frac{dx}{dt} = \frac{x + 2xt + \cos t}{1 + t^2}$$

(a) Separable but not linear

(b) Linear but not separable.

(c) Both separable and linear

(d) Neither separable nor linear.

11. Which of the following differential equations is with separated variables?

(a) $\log\left(\frac{dy}{dx}\right) = ax + by$

(b) $dy = (e^{x-y} + x^2 e^{-y}) dx$

(c) $\tan y dy = [\sin(x+y) + \sin(x-y)] dx$

☒ (d) All of the above equations

12. The geometrical interpretation of the graph of the differential equation: $\frac{dy}{dx} = -\frac{x}{y}$ is a family of

(a) Hyperbola

(b) Ellipse

(c) Parabola

(d) Circle

13. The integrating factor of $\frac{dy}{dx} + \frac{x}{1+x} y = 1+x$ is:

(a) e^x

(b) $\frac{e^x}{1+x}$

(c) $e^x(1+x)$

(d) $e^{x+x^2/2}$

14. Identify the functions $p(t)$ and $q(t)$ if the differential equation $\frac{dx}{dt} = \frac{x + t^2 - 2x\sqrt{t}}{t}$ is written in the form

$$\frac{dx}{dt} + p(t)x = q(t)$$

(a) It is not possible to write

(b) $p(t) = 2\sqrt{t} - 1, q(t) = t^2$

(c) $p(t) = 1 - 2\sqrt{t}, q(t) = t$

(d) $p(t) = \frac{2\sqrt{t} - 1}{t}, q(t) = t$

15. An integrating factor $I(x)$ is found for the differential equation $(1+x^2)\frac{dy}{dx} + xy = 0$ and the equation is

rewritten as $\frac{d}{dx}[I(x)y] = 0$. Which of the following option is correct.

(a) $I(x) = e^{x^2/2}$

(b) $I(x) = \sqrt{1+x^2}$

(c) $I(x) = 1+x^2$

(d) $I(x) = 0$



16. Find the general solution of the differential equation $\frac{dy}{dx} + \frac{x}{1+x}y = 1+x$
- (a) $(1+ce^{-x})(1+x)$ (b) $1+x+c$
- (c) $c(1+x)$ (d) $e^{-x}\left(x + \frac{x^2}{2} + c\right)(1+x)$
17. $\tan^{-1}x + \tan^{-1}y = c$ is the general solution of the differential equation
- (a) $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ (b) $\frac{dy}{dx} = \frac{1+x^2}{1+y^2}$
- (c) $(1+x^2)dy + (1+y^2)dx = 0$ (d) $(1-x^2)dx + (1-y^2)dy = 0$
18. Find the particular solution of differential equation $\frac{dy}{dx} = x \sin(3x^2)$ given $y(0) = 0$
- (a) $y = -\frac{\cos(3x^2)}{6}$ (b) $y = \frac{\cos(3x^2)-1}{6}$ (c) $y = -\frac{\cos(3x^2)}{6} + c$ (d) $y = \frac{1-\cos(3x^2)}{6}$
19. If f_1 and f_2 be the integrating factors of the differential equations $x\frac{dy}{dx} + 2y = 1$ and $\frac{dy}{dx} - \frac{2}{x}y = \frac{1}{x}$ respectively, then
- (a) $f_1 = x^4f_2$ (b) $f_1 = x^3f_2$ (c) $f_1 = x^2f_2$ (d) $f_1 = f_2$
20. Consider the following differential equation: $\frac{dy}{dx} - 2x = f(x); (x \in R)$, satisfying $y(0) = 0$;
- where $f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ 1 & \text{for } x > 0 \end{cases}$. This initial value problem
- (a) has a continuous solution which is not differentiable at $x=0$
- (b) has a continuous solution which is differentiable at $x=0$
- (c) has a continuous solution which is differentiable on 'R'
- (d) does not have a continuous solution on 'R'
21. Which of the following transformations reduces the differential equation $\frac{dy}{dx} - \frac{\tan y}{1-x} = (1+x)e^x \sec y$ into the form $\frac{dz}{dx} + P(x)z = Q(x)$, by the substitution
- (a) $z = \sin y$ (b) $z = \cos y$ (c) $z = \operatorname{cosec} y$ (d) $z = \tan y$
22. If $y(x)$ satisfies the differential equation $\frac{dy}{dx} + 2y = 2 + e^{-x^2}$ with $y(0) = 0$, then $\lim_{n \rightarrow \infty} y(x)$ is equal to
- (a) 0 (b) 1 (c) 2 (d) -1



23. Consider the system of differential equations: $\frac{dx}{dt} = -y, \frac{dy}{dt} = x$

The trajectory of the particle 'P' located at point (x,y) will be

- (a) hyperbola (b) parabola (c) circle (d) Straight line

24. From the solution of the differential equation: $\frac{dy}{dx} = \frac{1}{1+x^2}$

What can you say regarding the following series?

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$$

- (a) The series is divergent (b) The series converges to 1

- (c) The series converges to $\frac{\pi}{4}$ (d) The series converges to 0

25. Which of the following is general solution to $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 10y = 0$?

(a) $y = A e^{2x} + B e^{-5x}$

(b) $y = A e^{-2x} + B e^{5x}$

(c) $y = A e^{2x} + B e^{5x}$

(d) $y = A e^{-2x} + B e^{-5x}$

26. If $y = e^{2t}$ is a solution to $\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + ky = 0$, what is the value of k?

(a) 6

(b) 4

(c) -6

(d) 0

27. Which of the following is general solution to $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 13x = 0$?

(a) $x = A e^{5t} + B e^{-t}$

(b) $x = e^{2t} (A \cos 3t + B \sin 3t)$

(c) $x = e^{3t} (A \cos 3t + B \sin 3t)$

(d) $x = e^{-3t} (A \cos 2t + B \sin 2t)$

28. The general solution to $\frac{d^2z}{dt^2} + 6\frac{dz}{dt} + 9z = 0$ is z. Which of the following options are correct?

(a) As $t \rightarrow \infty, z \rightarrow A$ for any value of B

(b) The behaviour of z as $t \rightarrow \infty$ depends on A, B

(c) As $t \rightarrow \infty, z \rightarrow 0$ for any value of A and B

(d) As $t \rightarrow \infty, z \rightarrow \infty$ for any value of A and B.

29. The particular solution to $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} - 4y = 0$ satisfying $y(0) = 0$ and $y'(0) = 5$

(a) $y = e^t - e^{-4t}$

(b) $y = e^{-t} - e^{4t}$

(c) $y = e^{-4t} - e^t$

(d) $y = e^{4t} - e^{-t}$



30. Find a solution to $\frac{d^2x}{dt^2} + \frac{dx}{dt} - 2x = 0$, which satisfies $x(0) = 3$ and does not tend to ∞ (or $-\infty$) as $t \rightarrow \infty$
- (a) $x = 3e^{-t}$ (b) $4e^{-2t} - e^t = x$ (c) $x = 3e^{-2t}$ (d) $x = 2e^{-2t} + e^{-t}$
31. If e^{2x} and $x e^{2x}$ are particular solutions of a 2nd order differential equation (homogeneous) with constant co-efficients, then the equation is:
- (a) $y'' - 4y' + 4y = 0$ (b) $y'' - 5y' + 6y = 0$ (c) $y'' - 4y = 0$ (d) $y'' - 3y' + 2y = 0$
32. The general solution of the differential equation $\frac{d^5y}{dx^5} - \frac{3d^4y}{dx^4} + \frac{3d^3y}{dx^3} - \frac{d^2y}{dx^2} = 0$ is
- (a) $c_1 + c_2x + (c_3 + c_4x + c_5x^2)e^x$ (b) $c_1x + c_2x^2 + c_3e^x + c_4e^{2x} + c_5e^{3x}$
 (c) $(c_1 + c_2 + c_3 + c_4 + c_5)e^x$ (d) None of these
33. The particular integral of the differential equation $y'' + y' + 3y = 5\cos(2x + 3)$ is:
- (a) $2\cos(2x + 3) - \sin(2x + 3)$ (b) $2\sin(2x + 3) + \cos(2x + 3)$
 (c) $\sin(2x + 3) - 2\cos(2x + 3)$ (d) $2\sin(2x + 3) - \cos(2x + 3)$
34. The particular integral of $\frac{d^4y}{dx^4} - y = \cos x \cosh x$ is:
- (a) $y = -\frac{1}{5} \sinh x \cos x$ (b) $y = -\frac{1}{5} \cosh x \cos x$
 (c) $y = -\frac{1}{5} \sinh x \sin x$ (d) $y = -\frac{1}{5} \cosh x \sin x$
35. Which one of the following differential equations represents all circles with radius a ?
- (a) $1 + \left(\frac{dy}{dx}\right)^2 + \sqrt{a^2 - x^2} \frac{d^2y}{dx^2} = 0$ (b) $1 + \left(\frac{dy}{dx}\right)^2 + \sqrt{a^2 - y^2} \frac{d^2y}{dx^2} = 0$
 (c) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 + a^2 \left(\frac{d^2y}{dx^2}\right)^2 = 0$ (d) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2 \left(\frac{d^2y}{dx^2}\right)^2$
36. Find the general solution of differential equation $\frac{d^2y}{dx^2} = \frac{1 + \cos^3 x}{\cos^2 x}$
- (a) $y = \tan x + \sec x + C$ (b) $y = \ln|\cos x| + \sin x + D$
 (c) $y = \tan x \sec x + \cos x + Cx + D$ (d) $y = -\ln|\cos x| - \cos x + Cx + D$
37. The general solution of differential equation $4x^2y'' - 8xy' + 9y = 0$ is:
- (a) $C_1 e^{5x/2} + C_2 e^{-3x/2}$ (b) $C_1 e^{3x/2} + C_2 e^{-3x/2}$
 (c) $(C_1 + C_2 \log x)x^{3/2}$ (d) $C_1 x^{3/2} + C_2 x^{-3/2}$



38. The solution of the differential equation $yy' + y^2 - x = 0$, where c is a constant, is
 (a) $y^2 = x + ce^{-2x}$ (b) $y^2 = x + ce^{-2x} - \frac{1}{2}$ (c) $y^2 = x + ce^{-2x} + \frac{1}{2}$ (d) $y^2 = x + ce^{-2x} + 1$
39. The differential equation $\frac{d^2y}{dx^2} + y = 0$ satisfying the conditions $y(0) = 1, y(\pi) = 0$, has
 (a) a unique solution (b) infinite no. of solutions
 (c) no solution ✓ (d) none of these
40. Consider a forced harmonic oscillator which obeys the differential equation: $\frac{d^2y}{dt^2} + y = \sin t$
 Which of the following is the solution of the differential equation with the initial condition $y(0) = 0$?
 (a) $y(t) = 6 \sin t$ (b) $y(t) = 12 \sin t + \frac{t}{2} \cos t$
 (c) $y(t) = 12 \sin t - \frac{t}{2} \cos t$ (d) $y(t) = 6 \cos t - \frac{t}{2} \cos t$
41. The function $f(x) = x^2$ can be written in terms of Legendre Polynomials as
 (a) $\frac{2P_2(x) + P_0(x)}{3}$ (b) $\frac{2P_1(x) - P_2(x)}{3}$ (c) $\frac{P_2(x) - P_1(x)}{3}$ (d) $\frac{P_2(x) - P_0(x)}{3}$
42. The function $f(x) = x^2 - x$ can be written in terms of Legendre Polynomials as
 (a) $\frac{3P_2(x) - P_1(x) + P_0(x)}{2}$ (b) $\frac{2P_2(x) - 3P_1(x) + P_0(x)}{3}$
 (c) $\frac{2P_2(x) - P_1(x) - P_0(x)}{3}$ (d) $\frac{2P_2(x) - P_1(x) + P_0(x)}{3}$
43. The solution of the differential equation $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 6y = 0$ is
 (a) $P_n(x)$ (b) $P_1(x)$ (c) $P_2(x)$ (d) $P_3(x)$
44. The value of the integral $\int_{-1}^1 [P_3(x)]^2 dx$ is equal to
 (a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{2}{7}$ (d) $\frac{1}{5}$
45. The value of the integral $\int_{-1}^1 P_n(x) dx$ ($n \neq 0$) is equal to
 (a) 0 (b) 1 (c) $\frac{1}{2n+1}$ (d) $\frac{2}{n^2-1}$
46. The value of the integral $\int_0^1 P_{2n}(x) dx$ ($n \neq 0$) is equal to
 (a) 0 (b) 1 (c) $\frac{1}{2n+1}$ (d) $\frac{2}{2n+1}$



47. The value of the integral $\int_{-1}^1 P_n(x)(1-2xz+z^2)^{-\frac{1}{2}} dx$ is equal to
 (a) $\frac{2}{2n+1}$ (b) $\frac{2z^n}{2n+1}$ (c) $\frac{z^n}{2n+1}$ (d) 0
48. The expression $P'_{2n}(0)$ can be written as
 (a) 0 (b) 1 (c) $\frac{1}{2n}$ (d) $\frac{(-1)^n \cdot n!}{2^n}$
49. The bessen function $J_{-2}(x)$ can be written as
 (a) $J_1(x)$ (b) $\frac{d}{dx}(J_1(x))$ (c) $J_2(x)$ (d) $\int J_1(x)dx$
50. The expression $\frac{d}{dx}(x^3 J_3(x))$ can be simplified to
 (a) $x^2 J_2(x)$ (b) $x^3 J_4(x)$ (c) $x^3 J_2(x)$ (d) $-x^3 J_2(x)$
51. The bessel function $J_0(1)$ can be best described by the series:
 (a) $1 - \frac{1}{2^2} + \frac{1}{2^2 \cdot 4^2} - \frac{1}{2^2 \cdot 4^2 \cdot 6^2} + \dots$ (b) $1 + \frac{1}{2^2} + \frac{1}{2^2 \cdot 4^2} + \frac{1}{2^2 \cdot 4^2 \cdot 6^2} + \dots$
 (c) $1 + \frac{1}{2 \cdot 2^2} + \frac{1}{3! \cdot 2^2 \cdot 4^2} + \frac{1}{4! \cdot 2^2 \cdot 4^2 \cdot 6^2} + \dots$ (d) $1 - \frac{1}{1^2} + \frac{1}{1^2 \cdot 3^2} - \frac{1}{1^2 \cdot 3^2 \cdot 5^2} + \frac{1}{1^2 \cdot 3^2 \cdot 5^2 \cdot 7^2} - \dots$
52. The value of the integral $\int_0^1 x[J_1(x)]^2 dx$ is equal to
 (a) $J_2(1)^2$ (b) 0 (c) $\frac{1}{2}[J_2(1)]^2$ (d) $J_1(1)^2$
53. The bessel function $J_2(x)$ can be written as
 (a) $J_1(x) - J_0(x)$ (b) $2J_1(x) - J_0(x)$ (c) $\frac{2}{x}J_1(x) - J_0(x)$ (d) $\frac{2}{x^2}J_1(x) - J_0(x)$
54. The expression $2J'_1(x)$ can be simplified to
 (a) $J_0(x) - J_2(x)$ (b) $J_1(x) - J_0(x)$ (c) $J_0(x) - J_3(x)$ (d) $J_1(x) - J_3(x)$
55. The co-efficient of z^{-3} in the expansion of $e^{\frac{x}{2}\left(z - \frac{1}{z}\right)}$ is equal to
 (a) $J_3(x)$ (b) $J_3(0)$ (c) $J_3(-x)$ (d) $-J_3(x)$



CSIR Previous Year Questions

56. Let $p_n(x)$ (where $n=0,1,2,\dots$) be polynomial of order 'n' with real co-efficients defined in the

interval $2 \leq n \leq 4$. If $\int_2^4 p_m(x) p_n(x) dx = \delta_{mn}$, then

[CSIR June -2011]

(a) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(-3-x)$ (b) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{3}(3+x)$

(c) $p_0(x) = \frac{1}{2}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$ (d) $p_0(x) = \frac{1}{\sqrt{2}}$ and $p_1(x) = \sqrt{\frac{3}{2}}(3-x)$

57. The generating function $F(x,t) = \sum_{n=0}^{\infty} P_n(x) t^n$ for Legendre polynomials $P_n(x)$ is

$F(x,t) = (1-2xt+t^2)^{-1/2}$. The value of $P_3(-1)$ is

[CSIR Dec -2011]

(a) 5/2

(b) 3/2

(c) 1

(d) -1

58. Let $x_1(t)$ and $x_2(t)$ be two linearly independent solutions of the differential equation

$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + f(t)x = 0$. Let, $w(t) = x_1(t)\frac{dx_2(t)}{dt} - x_2(t)\frac{dx_1(t)}{dt}$. If $w(0) = 1$, then $w(1)$ is given by

[CSIR Dec -2011]

(a) 1

(b) e^2

(c) $\frac{1}{e}$

(d) $\frac{1}{e^2}$

59. Let $y(x)$ be a continuous real function in the range of 0 to 2π , satisfying the inhomogeneous differential

equation: $\sin x \frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} = \delta\left(x - \frac{\pi}{2}\right)$. The value of $\frac{dy}{dx}$ at the point $x = \frac{\pi}{2}$

[CSIR June -2012]

(a) is continuous

(b) has a discontinuity of 3

(c) has a discontinuity of 1/3

(d) has a discontinuity of 1

60. The graph of the function $f(x)$ shown below is best described by

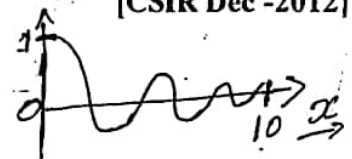
[CSIR Dec -2012]

(a) The bessel function $J_0(x)$

(b) $\cos x$

(c) $e^{-x} \cos x$

(d) $\frac{\cos x}{x}$



61. A function $f(x)$ obeys the differential equation $\frac{d^2f}{dx^2} - (3-4i)f = 0$ and satisfies the condition

$f(0) = 1$ and $f(x) \rightarrow 0$ as $x \rightarrow \infty$. The value of $f(\pi)$ is

[CSIR Dec -2012]

(a) $e^{2\pi}$

(b) $e^{-2\pi}$

(c) $-e^{-2\pi}$

(d) $-e^{2\pi}$

62. Given that $\sum_{n=0}^{\infty} H_n(x) \frac{t^n}{n!} = e^{-t^2+2tx}$, the value of $H_4(0)$ is

[CSIR June -2013]

(a) 12

(b) 6

(c) 24

(d) -6



63. The solution of the differential equation $\frac{dx}{dt} = x^2$ with the initial condition $x(0)=1$ will blow up as t tend to
 (a) 1 (b) 2 (c) $1/2$ (d) ∞ [CSIR June -2013]
64. Consider the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$ with initial conditions $x(0) = 0$ and $\dot{x}(0) = 1$. The solution $x(t)$ attains its maximum value when ' t ' is
 (a) $1/2$ (b) 1 (c) 2 (d) ∞ [CSIR June -2014]
65. Given, $\sum_{n=0}^{\infty} P_n(x)t^n = (1-2xt+t^2)^{-1/2}$ for $|t| < 1$, the value of $P_5(-1)$ is
 (a) 0.26 (b) 1 (c) 0.5 (d) -1 [CSIR June -2014]

ANSWER KEY

1.(a)	2.(d)	3.(c)	4.(b)	5.(a)	6.(b)	7.(b)	8.(b)	9.(c)	10.(b)
11.(c)	12.(d)	13.(b)	14.(d)	15.(b)	16.(a)	17.(c)	18.(d)	19.(a)	20.(a)
21.(a)	22.(b)	23.(c)	24.(c)	25.(a)	26.(a)	27.(b)	28.(c)	29.(d)	30.(c)
31.(a)	32.(a)	33.(d)	34.(b)	35.(d)	36.(d)	37.(c)	38.(b)	39.(c)	40.(c)
41.(a)	42.(b)	43.(d)	44.(c)	45.(a)	46.(a)	47.(b)	48.(a)	49.(c)	50.(c)
51.(a)	52.(c)	53.(c)	54.(a)	55.(d)	56.(d)	57.(d)	58.(d)	59.(d)	60.(a)
61.(c)	62.(a)	63.(a)	64.(b)	65.(d)					





SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

upto = 9

SECTION: MATHEMATICAL PHYSICS

Assignment 10: DIFFERENTIAL EQUATIONS

1. Solve the differential equations: (Separation of variables):

(i) $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$ given that $y(0) = \sqrt{3}$

(ii) $(e^y + 1)\cos x dx + e^y \sin x dy = 0$ given that $y\left(\frac{\pi}{3}\right) = 0$

(iii) $3 \tan y \cdot e^x dx + (1 - e^x) \sec^2 y dy = 0$ given that $y(1) = \frac{\pi}{4}$

(iv) $\frac{dy}{dx} = (4x + y + 1)^2$ given that $y(0) = 1$

(v) $x^4 \frac{dy}{dx} + x^3 y = -\sec(xy)$ given that $y(1) = \frac{\pi}{6}$

[Ans. (i) $\sqrt{1+x^2} + \sqrt{1+y^2} = 3$; (ii) $\sin x (e^y + 1) = \sqrt{3}$; (iii) $(1 - e^x)^3 = (1 - e)^3 \tan y$

(iv) $\tan^{-1}\left(\frac{4x+y+1}{2}\right) = 2x + \frac{\pi}{4}$; (v) $\sin(xy) = \frac{1}{2x^2}$]

2. Solve the differential equations: (Homogeneous type)

(i) $(y^2 - xy)dx + x^2 dy = 0$ given that $y(1) = 1$

(ii) $\left(y \sin \frac{y}{x} - x\right)dx - x \sin \frac{y}{x} dy = 0$ given that $y(4) = \pi$

(iii) $(1 + e^{x/y})dx + e^{x/y} \left(1 - \frac{x}{y}\right)dy = 0$ given that $y(0) = 1$

(iv) $x \frac{dy}{dx} = y \left(\log \frac{y}{x} + 1\right)$ given that $y(1) = 2$

[Ans. (i) $\frac{x}{y} = \ln x + 1$; (ii) $\cos\left(\frac{y}{x}\right) = \frac{1}{\sqrt{2}} - \ln 4 + \ln x$; (iii) $e^{x/y} + \frac{x}{y} = \frac{1}{y}$; (iv) $\ln \frac{y}{x} = x \ln 2$]

3. Solve the differential equations: (Linear type)

(i) $\cos^2 x \frac{dy}{dx} + y = \tan x$ given that $y\left(\frac{\pi}{4}\right) = 1$

(ii) $(1 + y^2)dx = (\tan^{-1} y - x)dy$ given that $x(1) = -1$

(iii) $(x^3 - x) \frac{dy}{dx} - (3x^2 - 1)y = x^5 - 2x^3 + x$ given that $y(e) = e$



(iv) $(1-x^2) \frac{dy}{dx} + 2xy = x(1-x^2)^{1/2}$ given that $y(0) = 1$

(v) $3 \frac{dy}{dx} + 3 \frac{y}{x} = 2x^4 y^4$ given that $y(1) = 1$

(vi) $\cos x \frac{dy}{dx} + 4y \sin x = 4\sqrt{y} \sec x$ given that $y\left(\frac{\pi}{4}\right) = 1$

(vii) $r \sin \theta - \frac{dr}{d\theta} \cos \theta = r^2$ given that $r\left(\frac{\pi}{4}\right) = 1$

[Ans. (i) $y = (\tan x - 1) + e^{(1-\tan x)}$; (ii) $x = \tan^{-1} y - 1 - \frac{\pi}{4} e^{\left(\frac{\pi}{4} - \tan^{-1} y\right)}$; (iii) $\frac{y}{x^3 - x} = \ln x - \frac{e^2}{e^2 - 1}$;

(iv) $y = \sqrt{1-x^2}$; (v) $\frac{1}{y^3} = -x^5 + 2x^3$; (vi) $\sec^2 x \cdot \sqrt{y} = 2 \left(\tan x + \frac{1}{3} \tan^3 x \right) - \frac{2}{3}$;

(vii) $\frac{1}{r} \sec \theta = \tan \theta + (\sqrt{2} - 1)$]

4. Solve the following differential equations: (exact equation)

(i) $(x^4 - 2xy^2 + y^4) dx - (2x^2y - 4xy^3 + \sin y) dy = 0$ given that $y(1) = 0$

(ii) $[1 + \ln(xy)] dx + \left[1 + \frac{x}{y}\right] dy = 0$ given that $y(1) = 1$

[Ans. (i) $\frac{x^5}{5} - x^2y^2 + xy^4 + \cos y = \frac{6}{5}$; (ii) $x \ln(xy) + y = 1$]

5. A inductance 'L' and resistance 'R' are connected in series with an e.m.f 'E'. If the current is zero at time $t=0$, find the current as a function of time. Also find the current at the end of 0.01 sec. If $E = 100$ Volts, $L = 100$ H, $R = 20$ Ohm.

6. A particle falls freely under gravity from a great height, there being a resistance proportional to the velocity of fall. Find the velocity and displacement as function of time.

7. The rate at which a hot body cools is proportional to the difference in temperature between it and surroundings (Newton's law of Cooling). A body is heated to 110°C and placed in air at 10° . Its temperature becomes 60° after 1 hour. After what additional time it cool down to 30° ?

8. In one dimensional steady state heat conduction for a hollow cylinder with thermal conductivity 'k' in the region $a \leq r \leq b$, the temperature T_r at a distance r ($a \leq r \leq b$) is given by $\frac{d}{dr} \left[r \frac{dT_r}{dr} \right] = 0$, subjected to the conditions $T_r = T_1$ at $r = a$ and $T_r = T_2$ at $r = b$. Determine the steady state temperature distribution T_r .

[Ans. (5) $i = \frac{E}{R} \left(1 - e^{-\frac{R}{L}t} \right)$; $i = 5(1 - e^{-0.002})$, (6) $x = \frac{gm}{k} \left(t + \frac{m}{k} (e^{-(k/m)t} - 1) \right)$;

(7) $\left(\frac{\ln 5}{\ln 2} - 1 \right) \text{hrs.}$, (8) $T_r = \frac{T_1 - T_2}{\ln \left(\frac{r_1}{r_2} \right)} \ln r + \frac{T_2 \ln r_1 - T_1 \ln r_2}{\ln \left(\frac{r_1}{r_2} \right)}$]



9. Solve the differential equations: (Homogeneous type):

(i) $\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$ given that $y(0) = -1, y'(0) = 0$

(ii) $\frac{d^2 y}{dx^2} + 6 \frac{dy}{dx} + 13y = 0$ given that $y(0) = 2, y'(0) = 1$

(iii) $\frac{d^2 y}{dx^2} - 8 \frac{dy}{dx} + 16y = 0$ given that $y(0) = 1, y'(0) = 2$

(iv) $\frac{d^3 y}{dx^3} - 2 \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} - 8y = 0$ given that $y(\infty) = 0, y'(0) = 2i$

[Ans. (i) $y = 2e^{3x} - 3e^{2x}$; (ii) $y = e^{-3x} \left(2 \cos 2x + \frac{7}{2} \sin 2x \right)$; (iii) $y = (1 - 2x)e^{4x}$; (iv) $y = -e^{-2ix}$]

10. Solve the differential equations: (Non-homogeneous type):

(i) $\frac{d^2 y}{dx^2} - \frac{dy}{dx} - 6y = e^x \cosh 2x$

(ii) $\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 2y = \sinh x$

(iii) $2 \frac{d^2 y}{dx^2} + 3 \frac{dy}{dx} + 4y = x^2 - 2x$

(iv) $(D^2 - 4D + 3)y = x^3$

(v) $\frac{d^2 y}{dx^2} + 4y = 2 \cos x \cos 3x$

(vi) $\frac{d^3 y}{dx^3} + y = \cos^2 \left(\frac{x}{2} \right) + e^{-x}$

(vii) $\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = x^3 e^{2x}$

(viii) $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = x e^x \sin x$

(ix) $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = x \sin x$

(x) $\frac{d^3 y}{dx^3} - 2 \frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = e^{3x}$

[Ans. (i) $y = c_1 e^{3x} + c_2 e^{-2x} + \frac{1}{10} x e^{3x} - \frac{1}{8} e^{-x}$; (ii) $y = e^{-x} (c_1 \cos x + c_2 \sin x) + \frac{1}{10} e^x + \frac{1}{2} e^{-x}$;

(iii) $y = e^{\frac{3}{4}x} \left(c_1 \cos \frac{\sqrt{23}}{4} x + c_2 \sin \frac{\sqrt{23}}{4} x \right) + \frac{1}{32} (8x^2 - 28x + 13)$; (iv) $y = c_1 e^x + c_2 e^{3x} +$

$\frac{1}{27} (9x^3 + 36x^2 + 78x + 80)$; (v) $y = c_1 \cos 2x + c_2 \sin 2x - \frac{1}{12} \cos 4x + \frac{x}{4} \sin 4x$;

(vi) $y = c_1 e^{-x} + e^{\frac{x}{2}} \left(c_2 \cos \frac{\sqrt{3}}{2} x + c_3 \sin \frac{\sqrt{3}}{2} x \right) + \frac{1}{2} + \frac{1}{4} (\cos x - \sin x + e^{-x})$;

(vii) $y = (c_1 + c_2 x) e^{2x} + e^{2x} \frac{x^5}{20}$; (viii) $y = (c_1 + c_2 x) e^x - e^x (x \sin x + 2 \cos x)$;

(ix) $y = (c_1 + c_2 x) e^x + \frac{1}{2} x \cos x + \frac{1}{2} \cos x - \frac{1}{2} \sin x$; (x) $y = c_1 e^x + c_2 e^{3x} + c_3 e^{-2x} + \frac{x}{10} e^{3x}$]



11. Solve the following differential equations:

(i) $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$

(ii) $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = \sin(\ln x^2)$

(iii) $\frac{d^2 y}{dx^2} + y = \sec x \tan x$

(iv) $\frac{d^2 y}{dx^2} + y = \frac{2}{1+e^x}$

(v) $\frac{dx}{dt} + 4x + 3y = t, \frac{dy}{dt} + 2x + 5y = e^t$

(vi) $\frac{dx}{dt} + 2y + \sin t = 0, \frac{dy}{dt} - 2x - \cos t = 0$

[Ans. (i) $y = c_1 x^4 + \frac{c_2}{x} + \frac{(\ln x)}{5} x^4$; (ii) $y = c_1 \cos(\ln x) + c_2 \sin(\ln x) - \frac{1}{3} \sin(\ln x^2)$;

(iii) $y = c_1 \cos x + c_2 \sin x + x \cos x + \sin x \ln(\sec x) - \sin x$;

(iv) $y = c_1 e^x + c_2 e^{-x} - 1 + e^x \ln(e^{-x} + 1) - e^{-x} \ln(e^x + 1)$;

(v) $x = c_1 e^{-2t} + c_2 e^{-7t} + \frac{5}{14}t - \frac{31}{196} - \frac{1}{8}e^t, y = -\frac{2}{3}c_1 e^{-2t} + c_2 e^{-7t} - \frac{1}{7}t + \frac{9}{98} + \frac{5}{24}e^t$;

(vi) $x = c_1 \cos 2t + c_2 \sin 2t - \cos t, y = c_1 \sin 2t - c_2 \cos 2t - \sin t$]

12. The radial displacement 'u' in a rotating disc at a distance 'r' from the axis is given by,

$$r^2 \frac{d^2 u}{dr^2} + r \frac{du}{dr} - u + kr^3 = 0 \quad (k \text{ is constant})$$

Find the displacement 'u' as a function of 'r' subject to the conditions $u(0) = 0, u(a) = 0$.

13. Check whether the following functions are linearly dependent or not?

(i) e^{ax}, e^{bx}

(ii) $e^{ax} \sin bx, e^{ax} \cos bx$

14. Find the singular points and discuss their nature for the following differential equation.

(i) $2x^2 y'' + 3xy' + (x^2 - 4)y = 0$

(ii) $x(x-2)^2 y'' + 2(x-2)y' + (x+3)y = 0$

15. Express the following polynomials into legendre polynomials:

(i) $4x^3 + 6x^2 + 7x + 2$

[Ans. $\frac{8}{5}P_3(x) + 4P_2(x) + \frac{47}{5}P_1(x) + 4P_0(x)$]

(ii) $4x^3 - 2x^2 - 3x + 8$

[Ans. $\frac{8}{5}P_3(x) - \frac{4}{3}P_2(x) - \frac{3}{5}P_1(x) + \frac{22}{3}P_0(x)$]

(iii) $x^4 + 2x^3 - 6x^2 + 5x - 3$

[Ans. $\frac{8}{35}P_4(x) + \frac{4}{5}P_3(x) - \frac{24}{7}P_2(x) + \frac{31}{5}P_1(x) - \frac{24}{5}P_0(x)$]

(iv) x^5

[Ans. $\frac{8}{63}P_5(x) + \frac{7}{9}P_3(x) + \frac{3}{7}P_1(x)$]

16. Prove that:

(i) $\sum_{n=0}^{\infty} P_n(x) = \frac{1}{\sqrt{2-2x}}$

(ii) $\frac{1-z^2}{(1-2xz+z^2)^{3/2}} = \sum_{n=0}^{\infty} (2n+1)P_n(x)$



$$(iii) \log \frac{1 + \sin \frac{\alpha}{2}}{\sin \frac{\alpha}{2}} = 1 + \frac{1}{2} P_1(\cos \alpha) + \frac{1}{3} P_2(\cos \alpha) + \frac{1}{4} P_3(\cos \alpha) + \dots$$

$$(iv) P'_{n+1} + P'_n = P_0 + 3P_1 + 5P_2 + \dots + (2n+1)P_n$$

$$(v) \int_{-1}^1 x^2 P_{n+1}(x) P_{n-1}(x) dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$$

$$(vi) \int_{-1}^1 (1-x^2) [P'_n(x)]^2 dx = \frac{2n(n+1)}{(2n+1)}$$

$$(vii) \int_{-1}^1 x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2-1}$$

17. Prove that,

$$(i) \sqrt{\frac{\pi x}{2}} J_{\frac{3}{2}}(x) = \frac{\sin x}{x} - \cos x$$

$$(iii) \frac{d}{dx} [J_n^2(x) + J_{n+1}^2(x)] = 2 \left[\frac{n}{x} J_n^2(x) - \frac{n+1}{x} J_{n+1}^2(x) \right]$$

$$(ii) \cos(x \sin \theta) = J_0 + 2J_2 \cos 2\theta + 2J_4 \cos 4\theta + \dots$$

$$\sin(x \sin \theta) = 2J_1 \sin \theta + 2J_3 \sin 3\theta + \dots$$

$$(iv) \frac{d}{dx} [x J_n(x) J_{n+1}(x)] = x [J_n^2(x) - J_{n+1}^2(x)]$$

18. Express the following polynomials as Hermite polynomials:

$$(i) 64x^4 + 8x^3 - 32x^2 + 40x + 10 \quad [\text{Ans. } 4H_4(x) + H_3(x) + 40H_2(x) + 26H_1(x) + 42H_0(x)]$$

$$(ii) 8x^3 + 8x^2 - 6x + 2 \quad [\text{Ans. } H_3(x) + 2H_2(x) + 3H_1(x) + 6H_0(x)]$$

19. Find the value of: (i) $\int_{-\infty}^{\infty} e^{-x^2} H_{20}(x) H_{10}(x) dx$ (ii) $\int_{-\infty}^{\infty} e^{-x^2} [H_4(x)]^2 dx$



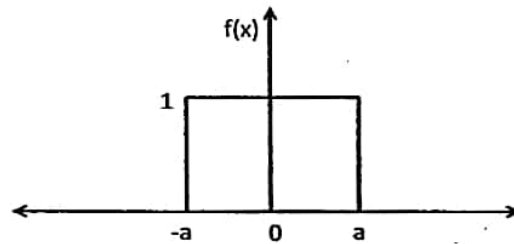


SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

SECTION: MATHEMATICAL PHYSICS

Assignment-10: FOURIER TRANSFORM

1. The fourier tranform of the following function will be



- (a) $\sqrt{\frac{2}{\pi}} \frac{\sin sa}{s}$ (b) $\sqrt{\frac{2}{\pi}} \frac{\cos sa}{s}$ (c) $\frac{1}{\sqrt{2\pi}} \frac{\cos sa}{s}$ (d) $\frac{1}{\sqrt{2\pi}} \frac{\sin sa}{s}$
2. The fourier transform of the function: $f(x) = 1 - x^2$ for $|x| \leq 1$
 $= 0$ for $|x| > 1$

- (a) $\frac{1}{\sqrt{2\pi}} \frac{4}{s^3} [-s \cos(s) + \sin(s)]$ (b) $\frac{1}{\sqrt{2\pi}} \frac{4}{s^3} [s \cos(s) + \sin(s)]$
 (c) $\frac{1}{\sqrt{2\pi}} \frac{4}{s^3} [s \sin(s) + \cos(s)]$ (d) $\frac{1}{\sqrt{2\pi}} \frac{4}{s^3} [s \sin(s) - \cos(s)]$

3. The fourier transform of the Gaussian wave packet given as:

$$f(x) = e^{-x^2/2} \text{ for } -\infty < x < \infty$$

will be

- (a) $\frac{1}{\sqrt{2\pi}} e^{-s^2/2}$ (b) $e^{-s^2/2}$ (c) $\sqrt{\frac{2}{\pi}} e^{-s^2}$ (d) $\sqrt{\frac{2}{\pi}} e^{-s^2/4}$
4. The fourier tranform of the function: $f(x) = e^{-a|x|}$ for $-\infty < x < \infty$ ($a > 0$)
- (a) $\sqrt{\frac{2}{\pi}} \frac{2}{a^2 - s^2}$ (b) $\frac{1}{\sqrt{2\pi}} \frac{2}{a^2 - s^2}$ (c) $\frac{1}{\sqrt{2\pi}} \frac{2a}{a^2 + s^2}$ (d) $\frac{1}{\sqrt{2\pi}} \frac{2a}{a^2 - s^2}$
5. The fourier transform of $f(x) = e^{-x^2+2x-1}$ ($-\infty < x < \infty$) will be

- (a) $e^{-\left(is + \frac{s^2}{2}\right)}$ (b) $e^{-\left(is - \frac{s^2}{2}\right)}$ (c) $\frac{1}{\sqrt{2}} e^{-\left(is - \frac{s^2}{4}\right)}$ (d) $\frac{1}{\sqrt{2}} e^{-\left(is + \frac{s^2}{2}\right)}$



6. If the fourier transform of $f(x)$ is $f(s)$, then the fourier transform of $\frac{d}{dx}[f(x)]$ will be
 (a) $sf(s)$ (b) $-isf(s)$ (c) $-2isf(s)$ (d) $2isf(s)$
7. The fourier tranform of the following function: $f(x) = e^{-s^2} \cos 2x$ ($-\infty < x < \infty$) will be
 (a) $\frac{1}{\sqrt{2}} e^{-\left(\frac{s^2+4}{4}\right)} \cosh(4s)$ (b) $\frac{1}{\sqrt{2}} e^{-\left(\frac{s^2+4}{4}\right)} \cosh(s)$
 (c) $\frac{1}{\sqrt{2}} e^{-(s^2-4)} \cosh(s)$ (d) $e^{-(s^2-4)} \sinh(2s)$
8. If the fourier transform of $f(x)$ is $f(s)$, then the fourier transform of $\frac{d^n}{dx^n}[f(x)]$ will be
 (a) $isf(s)$ (b) $-isf(s)$ (c) $(-is)^n f(s)$ (d) $(-is)^{n-1} f(s)$
9. Use the information of Q.1. and then the value of the following integral will be :

$$\int_0^{\infty} \left(\frac{\sin t}{t} \right)^2 dt$$

- (a) $\frac{\pi}{2}$ (b) π (c) 0 (d) 1
10. The value of the following integral:

$$\int_0^{\infty} \frac{dx}{(a^2+x^2)(b^2+x^2)}$$

- (a) $\frac{\pi}{2ab(a-b)}$ (b) $\frac{\pi}{2ab(a+b)}$ (c) $\frac{\pi}{2ab}$ (d) $\frac{\pi}{2(a+b)}$
11. The fourier transform of the dirac delta function $f(x) = \delta(x-a)$ will be
 (a) $\frac{1}{\sqrt{2\pi}}$ (b) $\frac{1}{\sqrt{\pi}}$ (c) $\frac{1}{\sqrt{2\pi}} e^{isa}$ (d) $\frac{1}{\sqrt{2\pi}} e^{sa}$

CSIR Previous Year Questions

12. Fourier Transform of the derivative of the dirac δ -function, namely $\delta'(x)$ is proportional to
 [CSIR Dec 2013]
 (a) 0 (b) 1 (c) $\sin k$ (d) ik

ANSWER KEY

1. (a) 2. (a) 3. (b) 4. (c) 5. (c) 6. (b) 7. (b)
 8. (c) 9. (a) 10. (b) 11. (c) 12. (d)





SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

SECTION: MATHEMATICAL PHYSICS

Assignment 11: LAPLACE TRANSFORM

1. Evaluate the Laplace transform of the following functions:

(i) $f(x) = (1 + \cos 2x)$

(ii) $f(x) = (x-1)$ if $1 < t < 2$
 $= (3-x)$ if $2 < t < 3$

(iii) $f(x) = 3e^{3x} + 5x^4 - 4\cos(3x)$

(iv) $f(x) = 2\sin 2x \cos 4x$

(v) $f(x) = e^{-x}(3\sinh 2x - 5\cosh 2x)$

(vi) $f(x) = (1 + xe^{-x})^3$

(vii) $f(x) = \cos^2 ax$

(viii) $f(x) = \sinh(ax) \cos(ax)$

(ix) $f(x) = e^{-ax} \frac{x^{n-1}}{(n-1)!}$

(x) $f(x) = (x^2 - 3x + 2)\sin 3x$

(xi) $f(x) = \sin \omega x$ if $0 < x < \frac{\pi}{\omega}$
 $= 0$ if $\frac{\pi}{\omega} < x < \frac{2\pi}{\omega}$

(xii) $f(x) = \frac{1}{\epsilon}$ if $0 \leq x \leq \epsilon$
 $= 0$ if $x > \epsilon$

(xiii) $f(x) = \frac{1 - e^x}{x}$

(xiv) $f(x) = \frac{\sin 2x}{x}$

Ans. (i) $\frac{(2s^2+4)}{s(s^2+4)}$, (ii) $\frac{1}{s^2}(e^{-s} - 2e^{-2s} + e^{-3s})$, (iii) $\frac{3}{s-3} + \frac{120}{s^5} - \frac{4s}{s^2+9}$, (iv) $\frac{6}{s^2+36} - \frac{2}{s^2+4}$,
 (v) $\frac{1-5s}{s^2+2s-3}$, (vi) $\frac{1}{s} + \frac{3}{(s+1)^2} + \frac{6}{(s+2)^3} + \frac{6}{(s+3)^4}$, (vii) $\frac{s^2+2a^2}{s(s^2+4a^2)}$, (viii) $\frac{a(s^2-2a^2)}{s^4+4a^4}$,
 (ix) $\frac{1}{(s+a)^n}$, (x) $-\frac{18s^2+54}{(s^2+9)^3} - \frac{18s}{(s^2+9)^2} + \frac{6}{(s^2+9)}$, (xi) $\frac{1}{1-e^{-\frac{2\pi s}{\omega}}}\left(\frac{\omega e^{-\frac{s\pi}{\omega}}}{s^2+\omega^2} + \omega\right)$,
 (xii) $\frac{1}{\epsilon s}(1 - e^{-\epsilon s})$, (xiii) $\ln \frac{s-1}{s}$, (xiv) $\cot^{-1} \frac{s}{2}$



2. Laplace transform of $f'(x)$ can be represented as
 (a) $sL[f(x)]$ (b) $sL[f(x)] - f(0)$ (c) $L[f(x)] - f(0)$ (d) $L[f(x)] - f(1)$
3. Laplace transform of $f''(x)$ can be represented as
 (a) $L^2[f(x)]$ (b) $s^2 L^2[f(x)] - f(0)$
 (c) $s^2 L[f(x)] - f'(0) - sf(0)$ (d) $s^2 L[f(x)]$
4. Laplace transform of $J_0(x)$ can be represented as
 (a) $\frac{1}{1+s^2}$ (b) $\frac{1}{\sqrt{1+s^2}}$ (c) $\frac{s}{\sqrt{1+s^2}}$ (d) $\frac{s}{1+s^2}$
5. Laplace transform of unit step function $u_a(x) = u(x-a) = 0$ if $x < a$ is given by,
 $= 1$ if $x \geq a$
 (a) e^{-sa} (b) $\frac{e^{-sa}}{s}$ (c) $\frac{e^{-sa}}{s^2}$ (d) e^{-2sa}
6. The value of the integral $\int_0^\infty xe^{-2x} \sin x \, dx$ is equal to
 (a) $\frac{4}{25}$ (b) $\frac{4}{50}$ (c) $\frac{2}{15}$ (d) $\frac{3}{35}$
7. The value of the integral $\int_0^\infty \frac{\sin x}{x} \, dx$ is equal to
 (a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π
8. If the error function is defined as: $\text{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2} \, dx$, then the laplace transform of $\text{erf}(\sqrt{t})$ is equal to
 (a) $\frac{1}{\sqrt{s(s+1)}}$ (b) $\frac{1}{\sqrt{(s+1)}}$ (c) $\frac{1}{s\sqrt{s(s+1)}}$ (d) $\frac{1}{s\sqrt{(s+1)}}$
9. The laplace transform of the following function:
 $f(x) = k[u_0(x) - 2u_a(x) + 2u_{2a}(x) - 2u_{3a}(x) + \dots]$
 (a) $\frac{k}{s} \tanh\left(\frac{as}{2}\right)$ (b) $\frac{k}{s} \tanh(as)$ (c) $\frac{k}{2s} \tanh(as)$ (d) $\frac{k}{2s} \tanh\left(\frac{as}{2}\right)$
10. The inverse laplace transform of $f(s) = \frac{s-2}{(s-2)^2 + 25} + \frac{s+4}{(s+4)^2 + 81} + \frac{1}{(s+2)^2 + 9}$ is
 (a) $e^{-2x} \cos 5x + e^{-4x} \cos 9x + \frac{e^{-2x}}{3} \sin 3x$ (b) $e^{2x} \cos 5x + e^{-4x} \cos 9x + \frac{e^{-2x}}{3} \sin 3x$
 (c) $e^{2x} \cos 5x + e^{4x} \cos 9x + \frac{e^{2x}}{3} \sin 3x$ (d) $e^{2x} \cos 5x + e^{4x} \cos 9x + \frac{e^{-2x}}{3} \sin 3x$

11. The inverse laplace transform of $f(s) = \frac{1}{(s-4)^5} + \frac{5}{(s-2)^2 + 25}$ is

- (a) $e^{4x} \frac{x^4}{12} - e^{2x} \cos 5x$ (b) $e^{4x} \frac{x^4}{12} + e^{2x} \cos 5x$
 (c) $e^{4x} \frac{x^4}{24} + e^{2x} \sin 5x$ (d) $e^{4x} \frac{x^4}{12} - e^{2x} \sin 5x$

12. The inverse Laplace transform of $f(s) = \frac{1}{s^2(s+1)^2}$ is

- (a) $te^{-t} + 2e^{-t} + t + 2$ (b) $te^{-t} + 2e^{-t} + t - 2$
 (c) $te^{-t} + e^{-t} + t - 2$ (d) $te^{-t} - e^{-t} + t - 2$

13. The inverse Laplace transform of $f(s) = \frac{3s+7}{s^2-2s-3}$ is

- (a) $2e^{3x} - e^{-x}$ (b) $4e^{3x} + e^{-x}$ (c) $2e^{3x} + e^{-x}$ (d) $4e^{3x} - e^{-x}$

CSIR Previous Year Questions

14. The graph of the function $f(x) = \begin{cases} 1 & \text{for } 2n \leq x \leq 2n+1 \\ 0 & \text{for } 2n+1 \leq x \leq 2n+2 \end{cases}$ (where $n=0,1,2,\dots$). Its

laplace transform $\tilde{f}(s)$ is

[CSIR Dec 2011]

- (a) $\frac{1+e^{-s}}{s}$ (b) $\frac{1-e^{-s}}{s}$ (c) $\frac{1}{s(1+e^{-s})}$ (d) $\frac{1}{s(1-e^{-s})}$

15. The inverse Laplace transform of $\frac{1}{s^2(s+1)}$ is

[CSIR June 2013]

- (a) $\frac{1}{2}t^2e^{-t}$ (b) $\frac{1}{2}t^2 + 1 - e^{-t}$ (c) $t - 1 + e^{-t}$ (d) $\frac{1}{2}t^2(1 - e^{-t})$

ANSWER KEY

- 2.(b) 3.(c) 4.(b) 5.(b) 6.(a) 7.(b)
 8.(d) 9.(a) 10.(b) 11.(c) 12.(b) 13.(d)
 14.(c) 15.(c)





SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

SECTION: MATHEMATICAL PHYSICS

Assignment 12: PARTIAL DIFFERENTIAL EQUATION

- The solution of the differential equation $2x \frac{\partial u}{\partial x} - 3y \frac{\partial u}{\partial y} = 0$ is
 (a) $Ax^{k/2}y^{k/3}$ (b) $Ax^{k/3}y^{k/2}$ (c) $Ax^{k/3}y^{k/3}$ (d) $Ax^{k/3}y^{k/4}$
- The solution of the differential equation $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ (Given that, $u(x, 0) = 6e^{-3x}$) is
 (a) $u(x, t) = 6e^{-3x+2t}$ (b) $u(x, t) = 6e^{-3x-3t}$ (c) $u(x, t) = 6e^{-3x-2t}$ (d) $u(x, t) = 6e^{-2x+3t}$
- The solution of the differential equation $4 \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3x$ (Given that, $u(x, 0) = 3e^{-x} - e^{-5x}$) is
 (a) $u(x, t) = 3e^{t-x} + e^{2t-5x}$ (b) $u(x, t) = 3e^{t-x} - e^{2t-5x}$
 (c) $u(x, t) = 3e^{t-x} + e^{-2t+5x}$ (d) $u(x, t) = 3e^{t-x} - e^{-2t-5x}$
- The vibration of a string of length 'L', fixed at both ends is described by the equation.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

subjected to the conditions $u(0, t) = u(L, t) = 0$, $u(x, 0) = 1$ and $\frac{\partial u}{\partial t}(x, 0) = 0$. The solution of the differential equation can be written as

- $\frac{2}{n\pi} [1 - (-1)^n] \sin \frac{n\pi x}{L} \cos \frac{n\pi ct}{L}$
 - $\frac{2}{n\pi} [1 - (-1)^n] \sin \frac{n\pi x}{L} \sin \frac{n\pi ct}{L}$
 - $\frac{2}{n\pi} [(-1)^n - 1] \sin \frac{n\pi x}{L} \cos \frac{n\pi ct}{L}$
 - $\frac{2}{n\pi} [1 - (-1)^n] \cos \frac{n\pi x}{L} \sin \frac{n\pi ct}{L}$
- The vibration of an elastic spring is governed by the partial differential equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$

The length of the spring is π and both ends are fixed. The initial velocity is zero and the initial deflection is $u(x, 0) = 2(\sin x + \sin 3x)$. The deflection $u(x, t)$ of the vibrating string will be

- $u(x, t) = 2[\cos 2t \sin x + \cos 3t \sin 3x]$
- $u(x, t) = 2[\cos t \sin x + \cos 3t \sin 3x]$
- $u(x, t) = 2[\sin t \sin x + \cos 3t \cos 3x]$
- $u(x, t) = 2[\sin t \sin x - \cos 3t \cos 3x]$



6. A string is stretched and fastened to two points distance 'l' apart. Motion is started by displacing the string into the form $y = kx(l-x)$ from which it is released at time $t = 0$. The displacement $y(x, t)$ at any time is written as

$$y(x, t) = \sum_{n=1}^{\infty} A_n \sin \frac{n\pi x}{L} \cos \frac{n\pi ct}{L}$$

The value of A_n is

- (a) 0 for odd n (b) 0 for even n (c) $\frac{8kl^2}{n^3\pi^3}$ for odd n (d) $\frac{8kl^2}{n^3\pi^3}$ for even n
7. The temperature distribution of a bar of length 2 units, (ends are kept at zero temperature) is governed by the following differential equation

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

subjected to the conditions $u(x, 0) = \sin\left(\frac{\pi x}{2}\right) + 3\sin\left(\frac{5\pi x}{2}\right)$

The temperature of the rigid bar will be

- (a) $\sin\left(\frac{\pi x}{2}\right)e^{-\pi^2 c^2 t/4} + 3\sin\left(\frac{5\pi x}{2}\right)e^{-25\pi^2 c^2 t/4}$ (b) $\sin\left(\frac{\pi x}{2}\right)e^{-\pi^2 c^2 t/4} + 3\sin\left(\frac{5\pi x}{2}\right)e^{-16\pi^2 c^2 t/4}$
 (c) $\sin\left(\frac{\pi x}{2}\right)e^{-\pi^2 c^2 t/4} + \cos\left(\frac{5\pi x}{2}\right)e^{-25\pi^2 c^2 t/4}$ (d) $\sin\left(\frac{\pi x}{2}\right)e^{-\pi^2 c^2 t/4} + 3\cos\left(\frac{5\pi x}{2}\right)e^{-25\pi^2 c^2 t/4}$
8. A rectangular plate with insulated surface is 8 cm. wide and its length can be treated as infinite compared to its width so that it may be considered as infinite in length. If the temperature along one short edge $y = 0$ is given by

$$u(x, 0) = 100 \sin\left(\frac{\pi x}{8}\right); (0 < x < 8)$$

The steady state temperature any plate will be

(Assume that all the edges of the rectangular plate are kept at 0°C)

- (a) $100 \sin\left(\frac{\pi x}{4}\right)e^{-\frac{\pi y}{4}}$ (b) $100 \cos\left(\frac{\pi x}{4}\right)e^{-\frac{\pi y}{4}}$ (c) $100 \sin\left(\frac{\pi x}{8}\right)e^{-\frac{\pi y}{8}}$ (d) $100 \cos\left(\frac{\pi x}{8}\right)e^{-\frac{\pi y}{8}}$
9. The solution of the differential equation:

$$\frac{\partial^2}{\partial t^2} u(x, t) - \frac{\partial^2}{\partial x^2} u(x, t) = 0$$

satisfying the boundary conditions $u(0, t) = 0 = u(L, t)$ and initial conditions

$$u(x, 0) = \sin\left(\frac{\pi x}{L}\right) \text{ and } \frac{\partial}{\partial t} u(x, t)|_{t=0} = \sin\left(\frac{2\pi x}{L}\right) \text{ is} \quad [\text{CSIR June 2013}]$$

(a) $\sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi t}{L}\right) + \frac{L}{2\pi} \sin\left(\frac{2\pi x}{L}\right) \cos\left(\frac{2\pi t}{L}\right)$



$$(b) 2 \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi t}{L}\right) - \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{2\pi t}{L}\right)$$

$$(c) \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{2\pi t}{L}\right) + \frac{L}{\pi} \sin\left(\frac{2\pi x}{L}\right) \sin\left(\frac{\pi t}{L}\right)$$

$$(d) \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi t}{L}\right) + \frac{L}{2\pi} \sin\left(\frac{2\pi x}{L}\right) \sin\left(\frac{2\pi t}{L}\right)$$

ANSWER KEY

- | | | | | | | |
|-------|-------|-------|-------|-------|------------|-------|
| 1.(a) | 2.(c) | 3.(b) | 4.(a) | 5.(b) | 6.(b), (c) | 7.(a) |
| 8.(c) | 9.(d) | | | | | |





SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

SECTION: MATHEMATICAL PHYSICS

Assignment-13: NUMERICAL TECHNIQUES

- Each of the following numbers are rounded-off upto three decimal places:
3.14750, 10.54761, 8.23450, 3.46756
Which of the following is true?
(a) $3.14750 \approx 3.148$, $10.54761 \approx 10.548$, $8.23450 \approx 8.235$, $3.46756 \approx 3.468$
(b) $3.14750 \approx 3.148$, $10.54761 \approx 10.548$, $8.23450 \approx 8.234$, $3.46756 \approx 3.468$
(c) $3.14750 \approx 3.147$, $10.54761 \approx 10.548$, $8.23450 \approx 8.234$, $3.46756 \approx 3.468$
(d) $3.14750 \approx 3.148$, $10.54761 \approx 10.547$, $8.23450 \approx 8.234$, $3.46756 \approx 3.468$
- If the number 9.0894 is rounded-off to three significant figures, then the round-off error is
(a) 0.0004 (b) -0.0004 (c) 0.001 (d) -0.001
- Consider the following series:
 $\sqrt{1} + \sqrt{2} + \sqrt{3} + \dots + \sqrt{100}$
If each of the roots are calculated upto rounding-off two decimal places, then the maximum possible error in the calculation of the sum of the series, is
(a) 0.05 (b) 0.5 (c) 5 (d) 50
- The interval for the smallest positive root of the transcendental equation $x = e^x$, is
(a) $[-1, 0]$ (b) $[-1, 1]$ (c) $[0, 1]$ (d) $[1, 2]$
- If the equation $x^3 - 3x + K = 0$ has a real root in the interval $[-1, 1]$, the range of K is
(a) $(-2, 2)$ (b) $(0, 2)$ (c) $(-2, 0)$ (d) $(1, 3)$
- The Newton raphson iterative formula to evaluate the square root of a natural number 'N' is
(a) $x_{n+1} = \frac{1}{2} \left(x_n + \frac{N}{x_n} \right)$ (b) $x_{n+1} = \frac{1}{2} \left(x_n - \frac{N}{x_n} \right)$
(c) $x_{n+1} = \frac{1}{2} \left(x_n + \frac{N}{2x_n} \right)$ (d) $x_{n+1} = \frac{1}{2} \left(x_n - \frac{N}{2x_n} \right)$
- The Newton raphson iterative formula to evaluate the cube root of a natural number 'N' is
(a) $x_{n+1} = \frac{3}{2} \left(x_n + \frac{N}{2x_n^2} \right)$ (b) $x_{n+1} = \frac{3}{2} \left(x_n - \frac{N}{2x_n^2} \right)$
(c) $x_{n+1} = \frac{2}{3} \left(x_n + \frac{N}{2x_n^2} \right)$ (d) $x_{n+1} = \frac{2}{3} \left(x_n - \frac{N}{2x_n^2} \right)$



8. Consider the following equation: $x^4 - x - 10 = 0$.
To solve the equation numerically using Newton-Raphson method, which one will be the best possible first guess value?
(a) 0 (b) 1 (c) $3/2$ (d) 2

9. The first order forward difference of $\ln[f(x)]$ is

(a) $\ln\left[1 + \frac{\Delta f(x)}{f(x)}\right]$ (b) $\ln\left[1 - \frac{\Delta f(x)}{f(x)}\right]$ (c) $\ln\left[1 - \frac{f(x)}{\Delta f(x)}\right]$ (d) $\ln\left[1 + \frac{f(x)}{\Delta f(x)}\right]$

10. The second order forward difference of $f(x) = \cos 2x$ is

(a) $4\sin^2 h \cos(2x + 2h)$ (b) $-4\sin^2 h \cos(2x + 2h)$
(c) $-4\cos^2 h \sin(2x + 2h)$ (d) $4\cos^2 h \sin(2x + 2h)$

Consider the following data:

x :	1	2	3	4
$f(x)$:	1	7	17	31

11. The value of $f(x)$ at $x=1.5$ is
(a) 2.5 (b) 3 (c) 3.5 (d) 4
12. The functional relationship between x and $f(x)$ for the above mentioned data, is
(a) $f(x) = x^2 - 1$ (b) $f(x) = 2x^2 - 1$ (c) $f(x) = 2x^2 + 1$ (d) $f(x) = x^2 + 1$
13. Consider the following data:

x :	0	1	2	3
$f(x)$:	0	1	3	10

The polynomial corresponding to the given data, is

(a) $\frac{2}{3}x^3 - \frac{3}{2}x^2 + \frac{11}{6}x$ (b) $\frac{2}{3}x^3 - \frac{3}{2}x^2 - \frac{11}{6}x$ (c) $\frac{2}{3}x^3 + \frac{3}{2}x^2 - \frac{11}{6}x$ (d) $\frac{2}{3}x^3 + \frac{3}{2}x^2 + \frac{11}{6}x$

14. Consider the following data:

x :	0	1	2	3	4	5	6
$f(x)$:	0	3	0	3	0	?	?

If the given data describes fourth order polynomial, then the missing terms are

(a) 45, 192 (b) 45, -192 (c) -45, 192 (d) -45, -192

15. Consider the following operators:

$$E[f(x)] = f(x+h); D[f(x)] = \frac{df}{dx}$$

Which of the following relation is true?

(a) $E = e^{hD}$ (b) $E = e^{-hD}$ (c) $E = 1 + e^{hD}$ (d) $E = 1 - e^{hD}$



16. Consider the following operators:

$$\delta[f(x)] = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right); \Delta[f(x)] = f(x+h) - f(x)$$

Which of the following relations is true?

(a) $\Delta = \frac{1}{2}\delta^2 - \delta\sqrt{1 + \frac{1}{4}\delta^2}$

(b) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}$

(c) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 - \frac{1}{4}\delta^2}$

(d) $\Delta = \frac{1}{2}\delta^2 - \delta\sqrt{1 - \frac{1}{4}\delta^2}$

17. The missing term in the following table:

x :	2	3	4	5	6
$f(x)$:	45.0	49.2	54.1	?	67.4

(a) 12.5

(b) 24.75

(c) 45.8

(d) 60.05

18. The missing values of the following table:

x :	45	50	55	60	65
$f(x)$:	3.0	?	2.0	?	-2.4

(a) 2.925, 0.225

(b) 2.525, 0.225

(c) 2.925, 0.775

(d) 2.525, 0.775

19. Consider the following data:

x :	-1	0	1	2	3
$f(x)$:	1	5	3	1	5

The value of the integral $\int_{-1}^3 f(x) dx$ is

(a) 8

(b) 10

(c) 12

(d) 16

20. Consider the following data:

x :	3	6	9	12
$f(x)$:	-1	1	2	3

The value of the integral $\int_3^{12} f(x) dx$ is

(a) 15

(b) 10

(c) 12

(d) 5

21. To evaluate $\int_{x_0}^{x_n} y dx$ by Simpson's 1/3 rd rule as well as Simpson's 3/8 th rule, the number of

intervals should be

(a) 2

(b) 3

(c) 6

(d) 10

22. To evaluate $\int_{x_0}^{x_n} y dx$ by Simpson's 1/3 rd rule as well as Trapezoidal rule, the number of

intervals should be

(a) 2

(b) 3

(c) 6

(d) 10



(110)
A second degree polynomial $f(x)$ has the following data:

$x:$	0	1	2
$f(x):$	1	4	15

23. The value of the integral $\int_0^2 f(x) dx$ (using Trapezoidal rule) will be
 (a) 8 (b) 12 (c) 16 (d) 20
24. The error in the estimation by trapezoidal rule, will be
 (a) $-4/3$ (b) $4/3$ (c) $5/3$ (d) $-5/3$
25. Consider the following differential equation: $\frac{dy}{dx} = -y$ given that, at $x=0, y=1$
 The value of y at $x=0.1$ (using Runge-kutta method of third order) is
 (a) 0.24 (b) 0.46 (c) 0.91 (d) 0.99
26. Consider the following differential equation: $\frac{dy}{dx} = x + y$ given that, at $x=0, y=1$
 The value of y at $x=0.2$ (using Runge-kutta method of fourth order) is
 (a) 0.94 (b) 1.11 (c) 1.24 (d) 1.67

ANSWER KEY

1.(b)	2.(d)	3.(c)	4.(c)	5.(a)	6.(a)
7.(c)	8.(d)	9.(a)	10.(b)	11.(c)	12.(b)
13.(a)	14.(d)	15.(a)	16.(b)	17.(d)	18.(a)
19.(c)	20.(c)	21.(c)	22.(a)	23.(b)	24.(a)
25.(c)	26.(c)				





SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

SECTION: MATHEMATICAL PHYSICS

Assignment 14: TENSOR ANALYSIS

- Which of the following is a tensor of order '0'?
(a) $\vec{A} + \vec{B}$ (b) $\vec{A} - \vec{B}$ (c) $\vec{A} \cdot \vec{B}$ (d) $\vec{A} \times \vec{B}$
- Which of the following statements is/are true?
(a) Gradient of scalar field is a contravariant tensor of rank 1
(b) Gradient of scalar field is a covariant tensor of rank 1
(c) Velocity is contravariant tensor of rank 1
(d) Velocity is contravariant tensor of rank 2
- If A is a scalar field, show that the second order derivatives of A i.e. $\frac{\partial^2 A}{\partial x^i \partial x^j}$ are not the components of a tensor.
- A covariant tensor has components $2x - z, x^2 y, yz$ in cartesian co-ordinate system. Find its components in cylindrical co-ordinate system.
[Ans. $2\rho \cos^2 \phi - z \cos \phi + \rho^3 \cos^2 \phi \sin^2 \phi; -\rho^2 \sin 2\phi + \rho z \sin \phi + \rho^4 \cos^3 \phi \sin \phi; \rho z \sin \phi$]
- The number of components for a tensor of rank 3 is
(a) 9 (b) 27 (c) 81 (d) 3
- The Krocnker delta tensor δ_i^j can be expressed as
(a) $\frac{\partial x^j}{\partial \bar{x}^k} \frac{\partial \bar{x}_k}{\partial x^i}$ (b) $\frac{\partial x^j}{\partial \bar{x}^k} \frac{\partial \bar{x}^k}{\partial x^i}$ (c) $\frac{\partial x^j}{\partial \bar{x}_k} \frac{\partial \bar{x}^k}{\partial x^i}$ (d) $\frac{\partial x^j}{\partial \bar{x}^k} \frac{\partial \bar{x}^k}{\partial x^i}$
- In contraction process,
(a) the rank of the tensor is reduced by 2.
(b) the rank of the tensor is reduced by 1.
(c) the rank of the tensor can be reduced by any number.
(d) the rank of the tensor is increased by 2.
- S_{ij} and A_{ij} represent a symmetric and anti-symmetric real valued tensor respectively in three dimension. The number of independent components of S_{ij} and A_{ij} are
(a) 3 and 6 (b) 6 and 3 (c) 6 and 6 (d) 9 and 6
- Let, $T_{ij} = \epsilon_{ijk} a_k$ and $\beta_k = \epsilon_{ijk} T_{ij}$ where ϵ_{ijk} is the Levi-Civita tensor. Then β_3 is equal to
(a) $2a_3$ (b) $-2a_3$ (c) a_3 (d) $-a_3$
- Which of the following is/are correct for Kroncker delta tensor δ_{ij} and Alternate tensor ϵ_{ijk} ?
(a) $\epsilon_{ijk} \delta_{ij} = 1$ (b) $\delta_{ij} \epsilon_{ijk} = 1$ (c) $\epsilon_{ijm} \epsilon_{klm} = \delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}$ (d) $\epsilon_{ijl} \epsilon_{kjl} = 2\delta_{ik}$



11. Find the metric tensor for the following:
 (a) a two dimensional plane in cartesian co-ordinate system.
 (b) a two dimensional plane in plane polar co-ordinate system.
 (c) three dimensional cartesian co-ordinate system.
 (d) three dimensional spherical polar co-ordinate system.
 (e) three dimensional cylindrical co-ordinate system.
12. If the line element 'ds' in a certain co-ordinate system is given by,

$$(ds)^2 = (dr)^2 + r^2 \sin^2 \theta (d\theta)^2 + r^2 \sin^2 \varphi (d\varphi)^2 + r \sin 2\varphi (dr d\theta)$$

then, the metric tensor will be

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 \sin^2 \theta & 0 \\ 0 & 0 & r^2 \sin^2 \varphi \end{bmatrix}$

(b) $\begin{bmatrix} 1 & r \sin 2\varphi & 0 \\ r \sin 2\varphi & r^2 \sin^2 \theta & 0 \\ 0 & 0 & r^2 \sin^2 \varphi \end{bmatrix}$

(c) $\begin{bmatrix} 1 & r \sin \varphi \cos \varphi & 0 \\ r \sin \varphi \cos \varphi & r^2 \sin^2 \theta & 0 \\ 0 & 0 & r^2 \sin^2 \varphi \end{bmatrix}$

(d) $\begin{bmatrix} 1 & \frac{r}{2} \sin \varphi \cos \varphi & 0 \\ \frac{r}{2} \sin \varphi \cos \varphi & r^2 \sin^2 \theta & 0 \\ 0 & 0 & r^2 \sin^2 \varphi \end{bmatrix}$

13. Consider a co-ordinate system (u, v, w) which is related to the cartesian co-ordinates by

$$x = vw, y = uv, z = uv$$

The metric tensor in terms of u, v, w is

(a) $\begin{bmatrix} v^2 + w^2 & uv & uv \\ uv & u^2 + w^2 & vw \\ uv & vw & u^2 + v^2 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & uv & uv \\ uv & 0 & vw \\ uv & vw & 0 \end{bmatrix}$

(c) $\begin{bmatrix} v^2 + w^2 & 0 & 0 \\ 0 & u^2 + w^2 & 0 \\ 0 & 0 & u^2 + v^2 \end{bmatrix}$

(d) None of these

14. If the line element 'ds' in a certain co-ordinate system is given by, $(ds)^2 = (dr)^2 + r^2 (d\theta)^2 + r^2 \sin^2 \theta (d\varphi)^2$

then find the christoffel symbols: (i) $[2, 2, 3]$, (ii) $[2, 2, 1]$, (iii) $\left\{ \begin{matrix} 1 \\ 2, 2 \end{matrix} \right\}$ [Ans. (i) 0, (ii) -r, (iii) -r]

15. Show that, $[i, k, j] + [j, k, i] = \frac{\partial g_{ij}}{\partial x^k}$

ANSWER KEY

1.(c)	2.(a), (c)	5.(b)	6.(b)	7.(a)	8.(b)
9.(a)	10.(c), (d)	12.(c)	13.(a)		





SUBJECT: PHYSICAL SCIENCES (NET-JRF/GATE)

SECTION: MATHEMATICAL PHYSICS

Assignment 15: GROUP THEORY

Consider the following set $\{1, -1, i, -i\}$

1. Which of the following statements is/are CORRECT?
 - (a) The set is a finite non-abelian group under multiplication.
 - (b) The set is a finite abelian group under multiplication.
 - (c) The set is a finite abelian group under addition.
 - (d) The set is a finite non-abelian group under addition.

Consider an element of a group as: $\begin{bmatrix} x & x \\ x & x \end{bmatrix}$ where x is a non-zero real number

2. The identity element of the group is

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

(d) $\begin{bmatrix} \frac{3}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{3}{2} \end{bmatrix}$

3. The inverse element of the group is

(a) $\begin{bmatrix} \frac{1}{x} & \frac{1}{x} \\ \frac{1}{x} & \frac{1}{x} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{4x} & \frac{1}{4x} \\ \frac{1}{4x} & \frac{1}{4x} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{2x} & \frac{1}{2x} \\ \frac{1}{2x} & \frac{1}{2x} \end{bmatrix}$

(d) $\begin{bmatrix} \frac{2}{x} & \frac{2}{x} \\ \frac{2}{x} & \frac{2}{x} \end{bmatrix}$

4. If a group is defined by $a * b = a + b - 1$, then the inverse of the group is
 - (a) $2 - a$
 - (b) a
 - (c) a^{-1}
 - (d) 0
5. In a group G , the elements are satisfying the relation $x^2 = E$ (identity element). Show that G is abelian in nature.
6. If G is an abelian group and $a, b \in G$ such that $O(a) = 5$ and $O(b) = 7$, then $(ab)^{14}$ is
 - (a) a
 - (b) a^{-1}
 - (c) a^2
 - (d) a^3
7. The generators of the following group:

$$G = \{a, a^2, a^3, a^4, a^5, a^6, a^7, a^8 = E\}$$

(a) a, a^2, a^4

(b) a^2, a^4, a^6

(c) a, a^3, a^5, a^7

(d) a^2, a^4, a^6, a^8



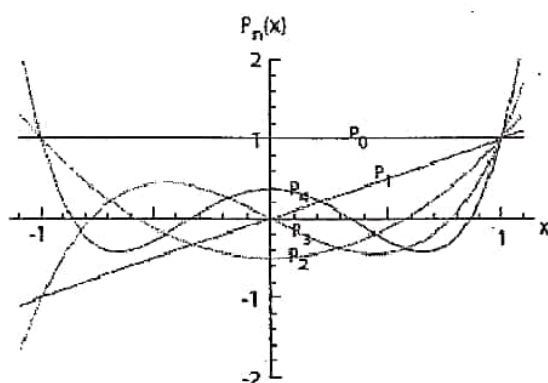
8. (i) Show that, the set of all n^{th} roots of unity forms a finite abelian group under multiplication.
(ii) Check that if it is cyclic or not. If yes, find its one generator.
9. The special unitary group of matrices i.e. $SU(2)$ can be represented by
- (a) $\begin{bmatrix} a & b \\ b^* & a^* \end{bmatrix}$ (b) $\begin{bmatrix} a & b \\ -b^* & a^* \end{bmatrix}$ (c) $\begin{bmatrix} a & b \\ -b^* & -a^* \end{bmatrix}$ (d) $\begin{bmatrix} a & b \\ b^* & -a^* \end{bmatrix}$
10. Which of the following matrices is an element of the group $SU(2)$?

(a) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1+i}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{1-i}{\sqrt{3}} \end{bmatrix}$ (c) $\begin{bmatrix} 2+i & i \\ 3 & 1+i \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

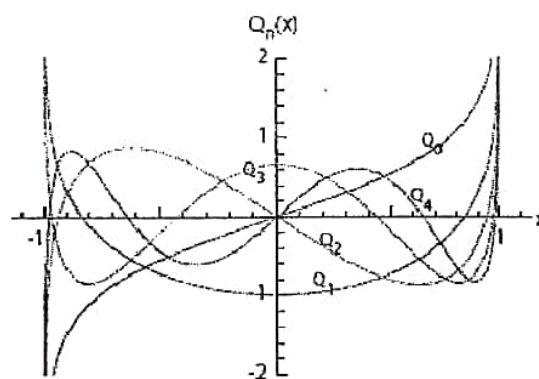
ANSWER KEY

- 1.(b) 2.(c) 3.(b) 4.(a) 6.(b) 7.(c)
- 9.(b) 10.(b)

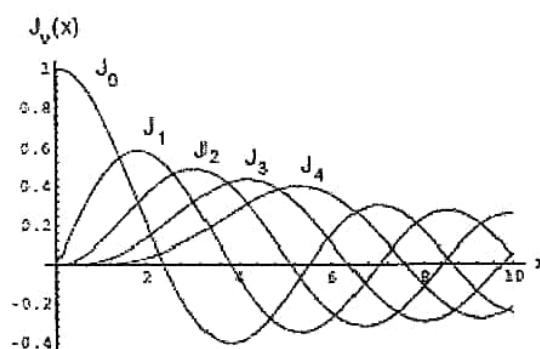




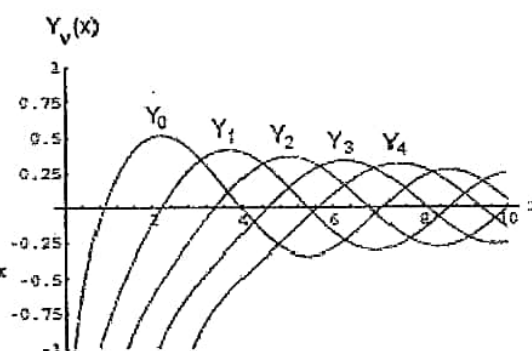
Legendre Polynomial



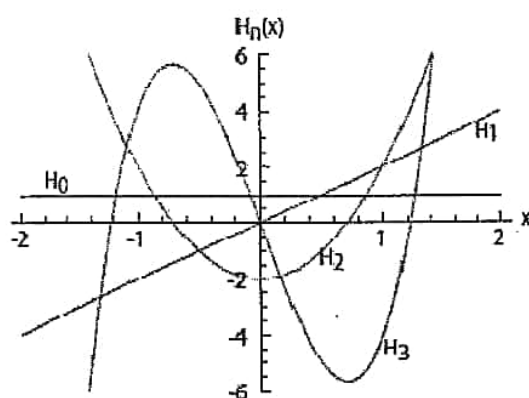
Legendre Function of 2nd Kind



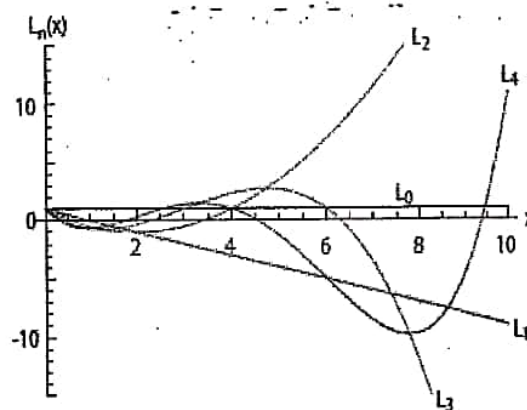
Bessel Function



Neumann Function



Hermite Polynomial



Laguarre Polynomial



PERMUTATIONS AND COMBINATIONS

- **Fundamental Principle of Counting:**

If an event can occur in 'm' different ways, following which another event will occur in 'n' different ways, then total number of ways in two events will occur in succession is ' $m \times n$ '.

Example:

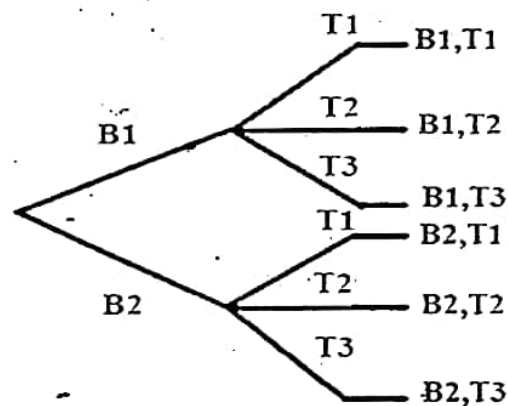
Q. Rahul has 2 school bags and 3 tiffin boxes. In how many ways can he carry these items?

Ans. Two events are following:

(A) The event of choosing a bag : It can be chosen in 2 ways.

(B) The event of choosing a tiffin box : It can be chosen in 3 ways.

So, Rahul can carry these items in $(2 \times 3) = 6$ ways which are following:



Q. Find the number of 4 letter words, which can be formed out of letters of the word 'ROSE' (Repetition is not allowed)

Ans. There are 4 vacant places. First place can be filled in 4 ways, second place can be filled in 3 ways, third place can be filled in 2 ways and fourth place can be filled in 1 way. (Since repetition is not allowed).

The total number of ways in which 4 places can be filled is $(4 \times 3 \times 2 \times 1) = 24$.

If repetition is allowed, then each of the 4 places can be filled in 4 ways. Then, total number of ways in which 4 places can be filled is $(4 \times 4 \times 4 \times 4) = 256$.

Q. How many 2 digit even numbers can be formed from digits 1,2,3,4,5 if digits can be repeated?

Ans. There are 2 vacant places. Since, the required number is even, then, the unit's place can be filled in 2 ways (either 2 or 4). The ten's place can be filled in 5 ways. Therefore, the required number of two digit numbers is $(2 \times 5) = 10$.

Exercise:

Q. A coin is tossed 3 times. How many possible outcomes are there? (Ans. 8)

Q. There are 11 different buses between Place 'A' and Place 'B'. In how many different ways a person can go from 'A' to 'B', if he travels with different buses in going and returning? (Ans. 110)

Q. How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each telephone number starts with 67 and no digit appears more than once?

(Ans. 336)

- **Permutations:**

A permutation is an arrangement of objects (taking some or all at a time) in a definite order.

Example: We consider the number of ways in which we can arrange the A,B,C in a row. There are 6 different ways: ABC, ACB, BAC, BCA, CAB, CBA. Each arrangement is called permutation of 3 different letters taken all at a time.

The code of a locker is '472'. If someone presses '742' it will not open. In this case order is important. That is 'Permutation'.

Theorem 1: The number of permutations of 'n' different objects taken 'r' ($0 \leq r \leq n$) at a time (the objects do not repeat) is denoted by ${}^n P_r$.

Proof : The possible number of permutations of 'n' things taken 'r' at a time is equivalent the number ways of filling 'r' things in 'r' positions. The 1st place can be filled in 'n' ways, the 2nd place can be placed in 'n-1' ways, ... the rth place can be filled in $[n-(r-1)]$ ways. Therefore,

$$\begin{aligned} {}^n P_r &= n(n-1)(n-2)\dots\dots\dots(n-r+1) \\ &= \frac{n(n-1)(n-2)\dots\dots\dots(n-r+1)(n-r)(n-r-1)\dots\dots\dots 3.2.1}{(n-r)(n-r-1)\dots\dots\dots 3.2.1} \\ &= \frac{n!}{(n-r)!} \end{aligned}$$

- ✓ The number of permutations of 'n' different objects taken 'n' at a time (the object do not repeat) is ${}^nP_n = n!$.

Q. In how many ways can the letters of the word "FORMULA" be arranged?

Ans. "FORMULA" contains 7 different letters. Total number of arrangements is $7! = 5040$.

Q. How many numbers (no digit is repeated) between 1000 and 10000 can be formed with 7,6,5,4,3,2.

Ans. Each number contains 4 digits. The required number = no. of permutations of 6 digits, taking 4 digits at a time = ${}^6P_4 = \frac{6!}{(6-4)!} = 6 \times 5 \times 4 \times 3 = 360$.

Q. If the letters of the word "MEANS" are arranged taken all at a time, how many numbers do not start with 'AS'?

Ans. Total number of arrangements = $5! = 120$

The numbers which starts with 'AS', the first two places can be filled in 1 way.

Then, the remaining 3 letters can be arranged in $3! = 6$ ways

The numbers which start with 'AS' = $1 \times 6 = 6$

The numbers which does not start with 'AS' = $120 - 6 = 114$

Exercise:

Q. How many numbers can be formed between 5000 and 6000 (no digit is repeated) using the digits 1,2,3,4,5,6,7,8? (Ans. 210)

Q. How many arrangements of the letters of the word "COINS" can be made, without changing the places of the vowels in the word? (Ans. 6)

Q. Find the number of arrangements of the letters of the word "STRANGE" can be made so that the vowels may be only in odd places. (Ans. 1440)

Q. If all the letters A,B,C,D,E,F,G can be arranged, find the number of the arrangements in which (i) A,B,C are all together, (ii) A,B,C are not all together. (Ans. (i) 720, (ii) 4320)

Theorem 2: The number of permutations of 'n' different objects taken 'r' at a time, (where repetition is allowed) is n^r .

Theorem 3: The number of permutations of 'n' things taken all at a time, where 'r' are of first kind, 's' are of second kind, ... and the rest are different will be

$$\frac{n!}{r!s!.....}$$

Q. Find the number of ways in which the letters of the word "INFINITESIMAL" can be arranged.

Ans. There are 13 letters among which 4 'I' and 2 'N' are present and the rest are different. Therefore the required number of permutations = $\frac{13!}{4!2!} = 129729600$

Q. If the letters of the word "CONSONANT" are arranged all at a time, find the number of arrangements in which (i) the two 'O's are together, (ii) three 'N's will be first.

Ans. (i) If the two 'O's are regarded as one thing, then there are 8 letters.

Required number of arrangements = $\frac{8!}{3!} = 6720$

(ii) If three 'N's are in first three places, other 6 letters can be arranged in

$\frac{6!}{2!} = 360$ ways.

- **Combinations:**

A combination is an arrangement of objects (taking some or all at a time) where order does not matter.

Example: Let us assume there is group of 3 Cricketers X, Y, Z. A team of 2 cricketers is to be formed. There are 3 possible teams that can be constructed i.e XY, YZ, XZ. Here, the team of X and Y is not different from Y and X i.e order is not important. Here each selection is called a Combination of 3 different objects taken 2 at a time.

Theorem 4: The number of combinations of 'n' different objects taken 'r' at a time

$$\text{is } {}^nC_r = \frac{n!}{r!(n-r)!}$$

Q. There are 5 teachers and 10 students. In how many ways can a committee of 3 teachers and 2 students be formed?

Ans. 3 teachers can be selected from 5 teachers in ${}^5C_3$ ways.

2 students can be selected from 10 students in ${}^{10}C_2$ ways.

$$\text{The required number of ways} = {}^5C_3 \times {}^{10}C_2 = \frac{5!}{3!2!} \times \frac{10!}{2!8!} = 450$$

Q. If a word consists of 5 vowels and 15 consonants, how many words of 3 different vowels and 2 different consonants can be made?

Ans. 3 vowels can be selected from 5 vowels in ${}^5C_3$ ways.

2 consonants can be selected from 15 consonants in ${}^{15}C_2$ ways.

3 vowels and 2 consonants can be selected in ${}^5C_3 \times {}^{15}C_2 = 1050$ ways.

5 letters can be arranged among themselves in $5!$ ways.

Hence, number of words = $1050 \times 120 = 126000$

Q. What is the number of ways of choosing 4 cards from a pack of 52 cards?

In how many ways of these

(i) Four cards are of same suit?

(ii) Four cards are of different suits?

(iii) Two cards are ace and two cards are queen?

(iv) Two cards are red and two cards are black?

(v) Four cards are of same colour?

Ans. There are four suits: Diamond, club, Spade, Heart (13 each). The number of ways in which 4 cards can chosen from 52 cards = ${}^{52}C_4$.

(i) Four cards will be of same suit i.e either 4 diamonds or 4 clubs or 4 spades or 4 hearts.

Required number of ways = ${}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 = 2890$

(ii) Four cards will be of different suits i.e. 1 diamond, 1 club, 1 spade and 1 heart.

Required number of ways = ${}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 = 13^4$

(iii) Required number of ways = ${}^4C_2 \times {}^4C_2 = 36$

(iv) Required number of ways = ${}^{26}C_2 \times {}^{26}C_2 = 105625$

(v) Required number of ways = ${}^{26}C_4 + {}^{26}C_4 = 29900$

Exercise:

Q. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) no girl? (ii) at least one boy and one girl? (iii) at least 3 girls? (Ans. (i) 21, (ii) 441, (iii) 91)

Q. In how many ways can 5 girls and 3 boys be seated in row so that no two boys are together? (Ans. 14400)